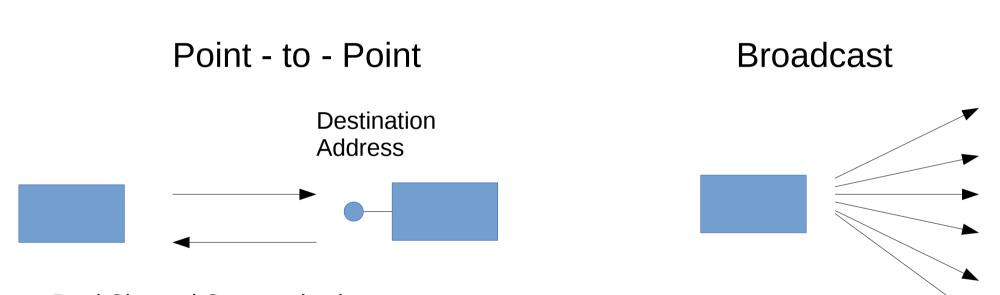
#### Introduction to Cryptography

arnaud.nauwynck@gmail.com

#### Communications



Dual Channel Communication ...example for Request - Response

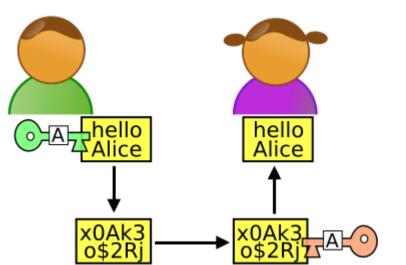
## Who are Alice, Bob, Eve, Malory, & Trent .. ?

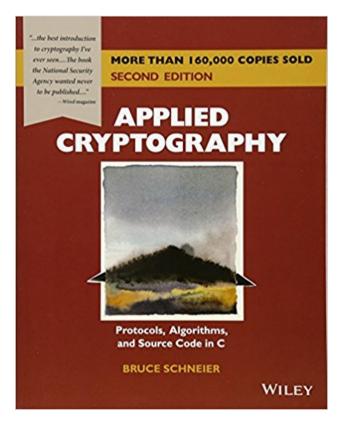
Article	Talk	Read	Edit	View history	Search Wikipedia	Q
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#### Alice and Bob

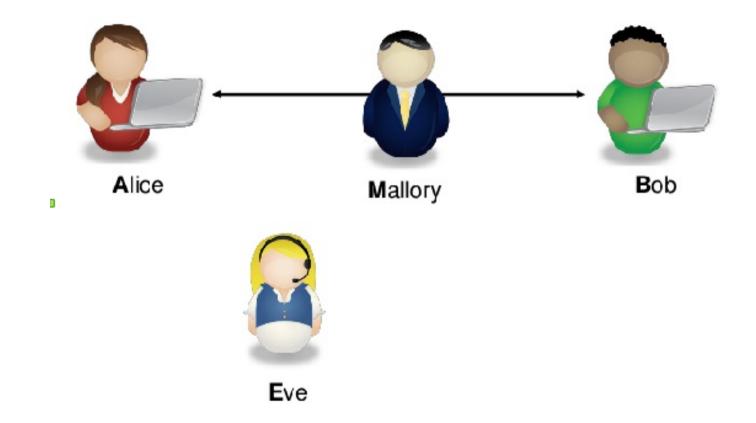
From Wikipedia, the free encyclopedia

Alice and Bob are fictional characters commonly used in cryptology, as well as science and engineering literature. The Alice and Bob characters were invented by Ron Rivest, Adi Shamir, and Leonard Adleman in their 1978 paper "A method for obtaining digital signatures and public-key cryptosystems."<sup>[1]</sup> Subsequently, they have become



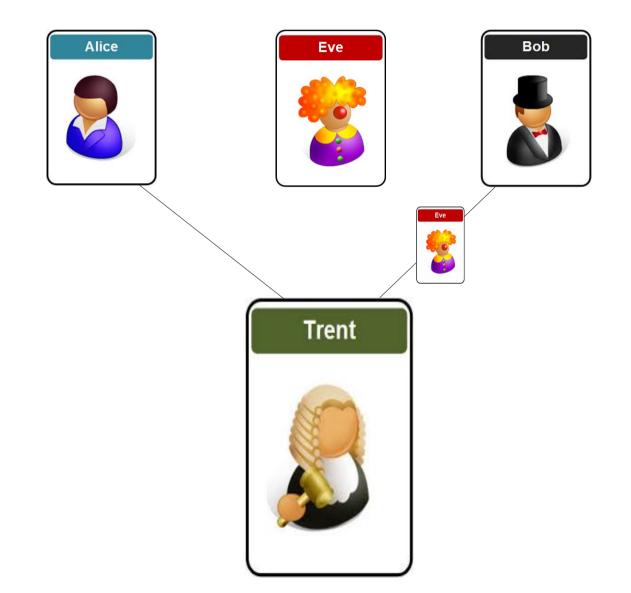


## Eve (E=Environment ... or Evil) and Malory (M=Malicious)



Mallory intercept + modify data ... Eve does't not

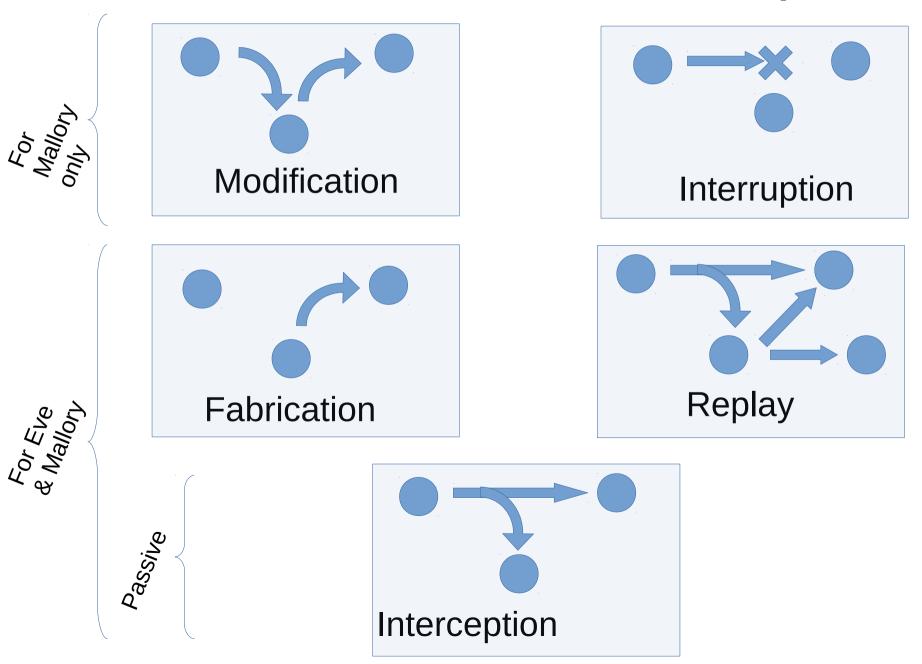
## In Trent you (may) Trust



#### M = Malory = Malicious User Man-In-The-Middle



#### Man-In-The-Middle Weapons



## A (Secured?) Chat



Allo Bob ? Alice speaking

Hi Alice



I have a secret to tell you I will tell you only if ...

Ok, I will do/pay/.. it

Secret = bla bla

## What's (Possibly) Wrong ?

Be Paranoiac !!

By Default, Everything is wrong

Proofs next ...

# Possibly Wrong

Authentication

Data Leak

Confidentiality

Integrity

Authorisation

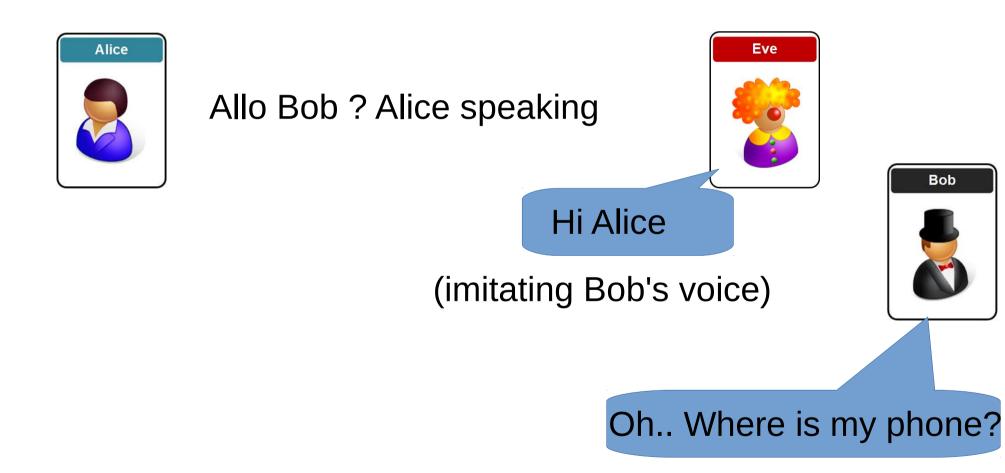
Password Crack

Data Replay

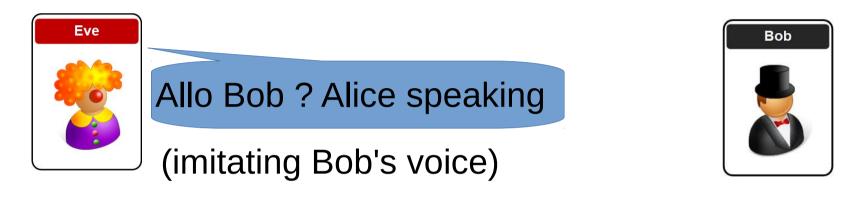
. . .

Predicatable guess

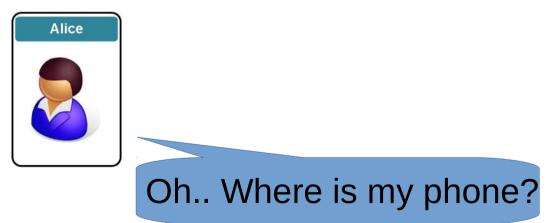
## Authentication (Receiver)



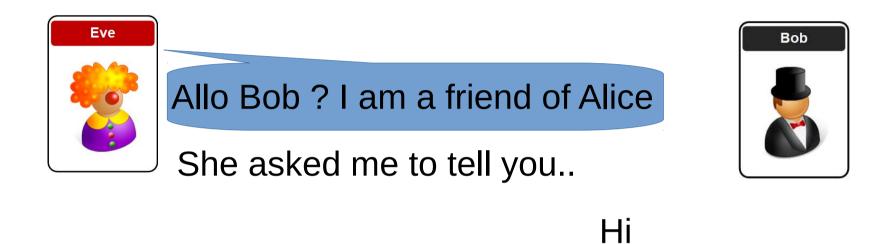
## Authentication (Sender)



**Hi Alice** 



#### ... Social Engineering



Sure, go ahead



#### Bob is so naive

#### Authentication denied



#### Allo Bob ? Alice speaking

Your voice is strange



#### **Basic Authentication**

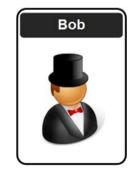


Allo Bob ? Alice speaking proof I am Alice: my password is ..





Allo Bob ? Alice speaking proof I am Alice: my password is ..



Hi Alice Your password is OK

#### **Password Challenge**



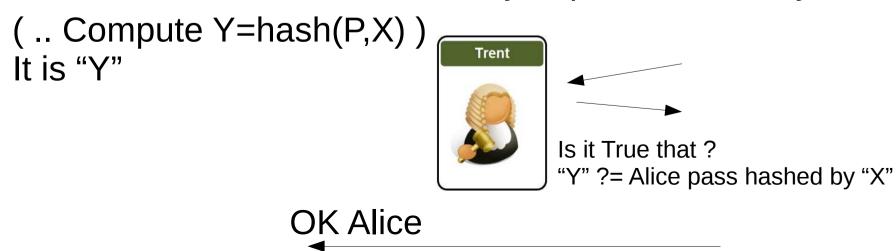
Allo Bob ? Alice speaking



Really ?

I don't give my password I can give you only a clue you can check with Trent

OK, Send me you pass hashed by "X"



# Trusted Thirdparty...



Allo Bob ? Alice speaking

Really ? Proove me your identity



Please Attest I am Alice (here is my password)

**OK** Alice



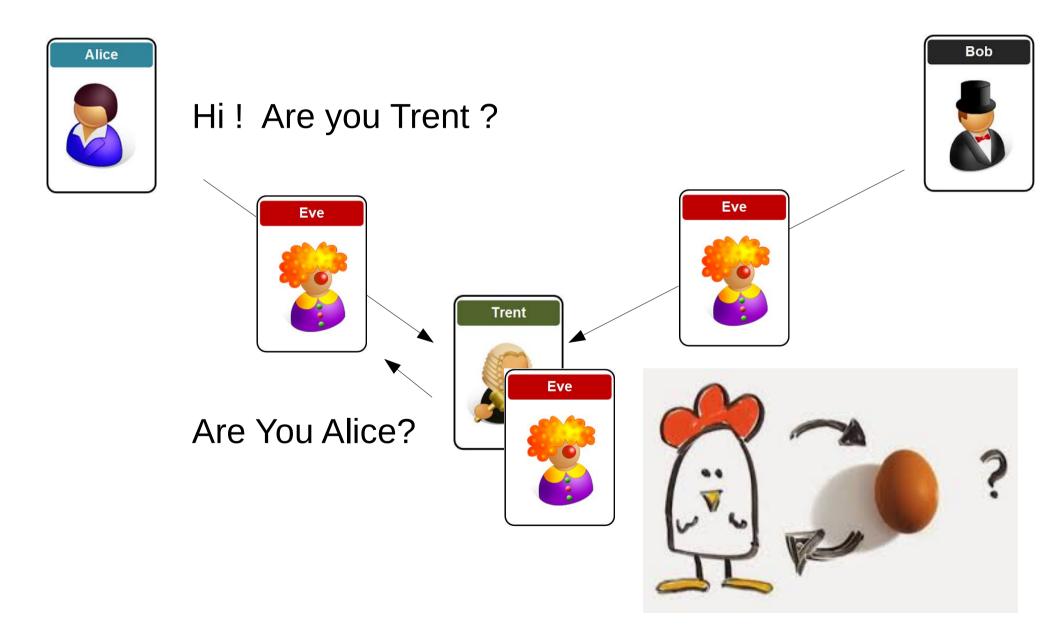
I ("Trent") certify this token paper was delivered to Alice on jun 2017: XADS4FSQ3RTXF

Bob: Ask "Trent" ... he prooved my identity by token XADS4FSQ3RTXF

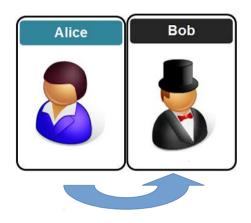
Is it true this token was given to Alice: XADS4FSQ3RTXF ?

Yes (And Alice did not complain yet being stolen)

## Problems With Thirdparties...



# Physical Hand-Shake ... Exchange Shared Key



Bob, here is a secret "key" for talking to you in 6 monthes

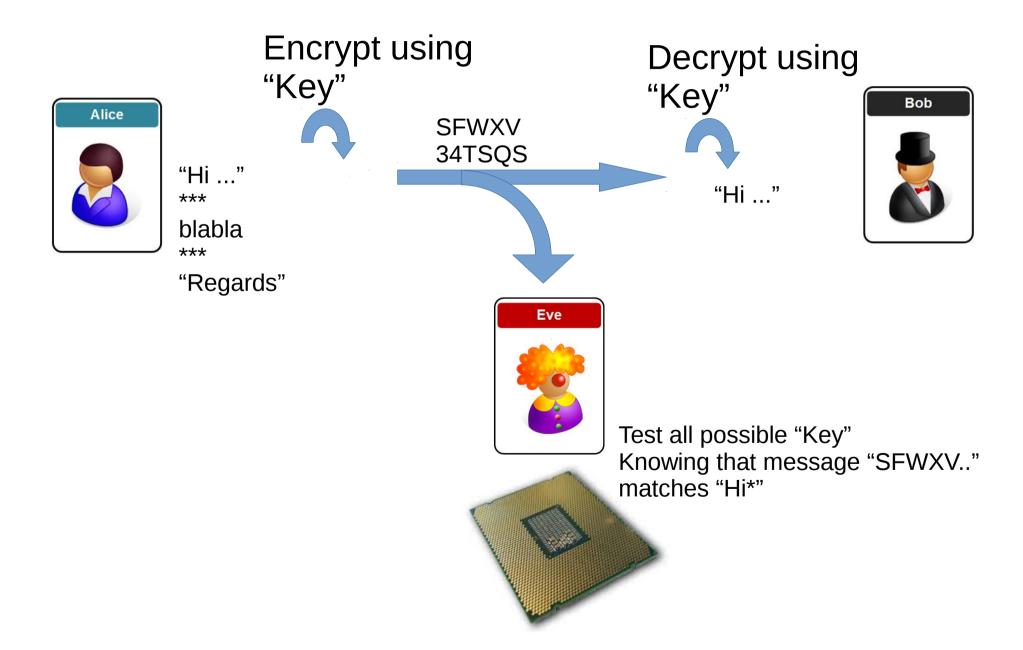


Hi Bob Let us use our private shared key (remember I gave you last 6 monthes)

\*\* Switch Encrypted \*\* SDFG23456RGQEST43



# "Key" recomputed from Data ...



#### "Key" inversion from Predicatable Data

WARN : do NOT use TOO many data ... AND NO predicatable data ...

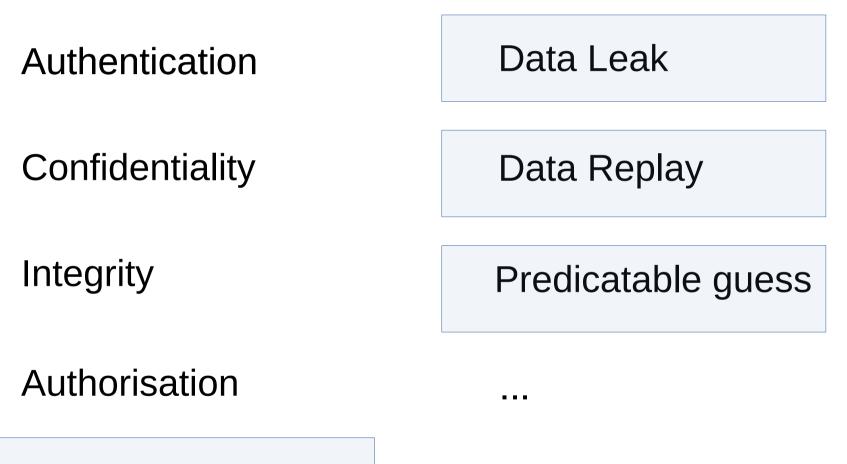
otherwise "Key" could be recomputed by Eve...

Thow "Key" at end of communication

Famous during World War 2 :

Alan Turing decrypted German Submarine "Enigma" Messages ... partly because all messages ended with "Hi Hitler"

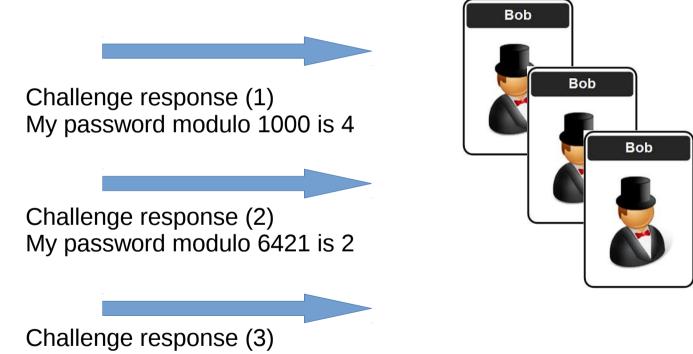
#### Encrypted ... & Still Weak



Password Crack

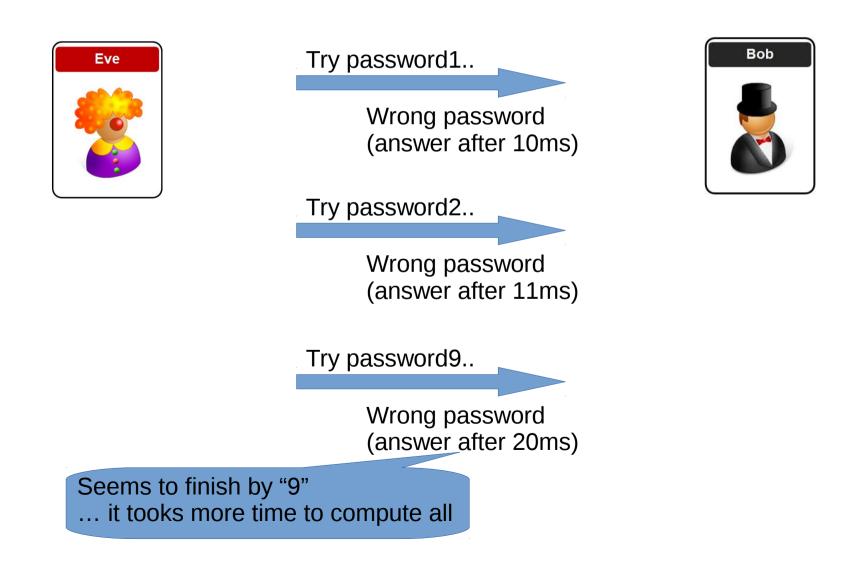
## (Partial) Data Leak Example : Modulo - Hashes





My password modulo 9482 is 7

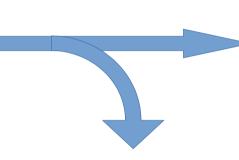
## Partial Data Leak Padding – Delay...

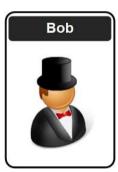


# Encrypted ... Data Replay

Encrypted credit card number: "XDFGZFG"





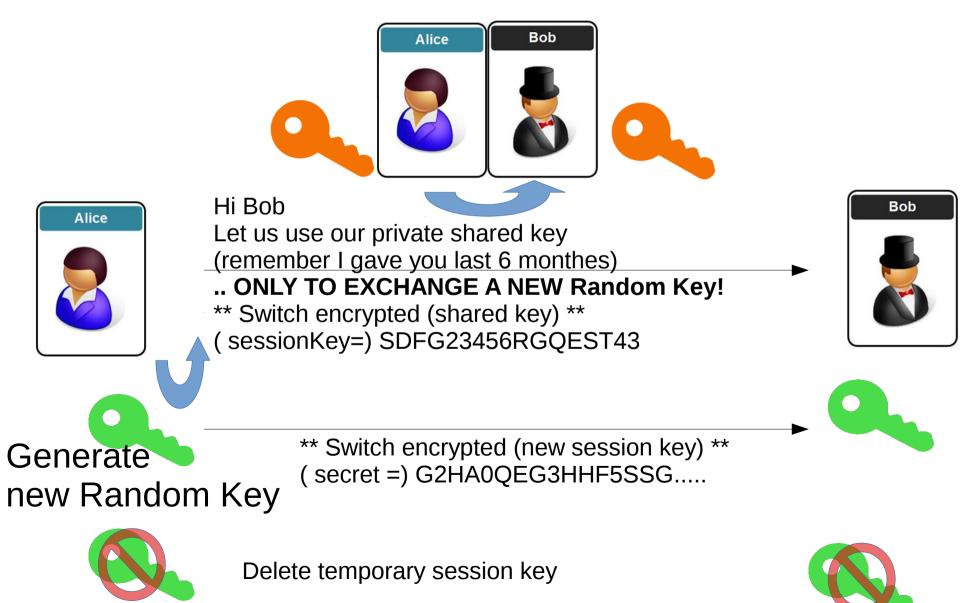




Encrypted credit card number: "XDFGZFG"



# Using Shared Key for Exchanging TMP SessionKey ONLY



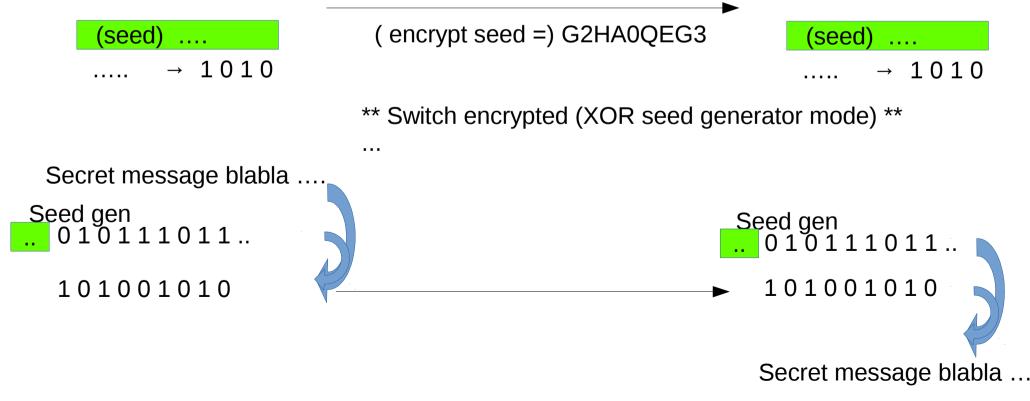
# Fast Crypt using XOR + Session Key Random Generator

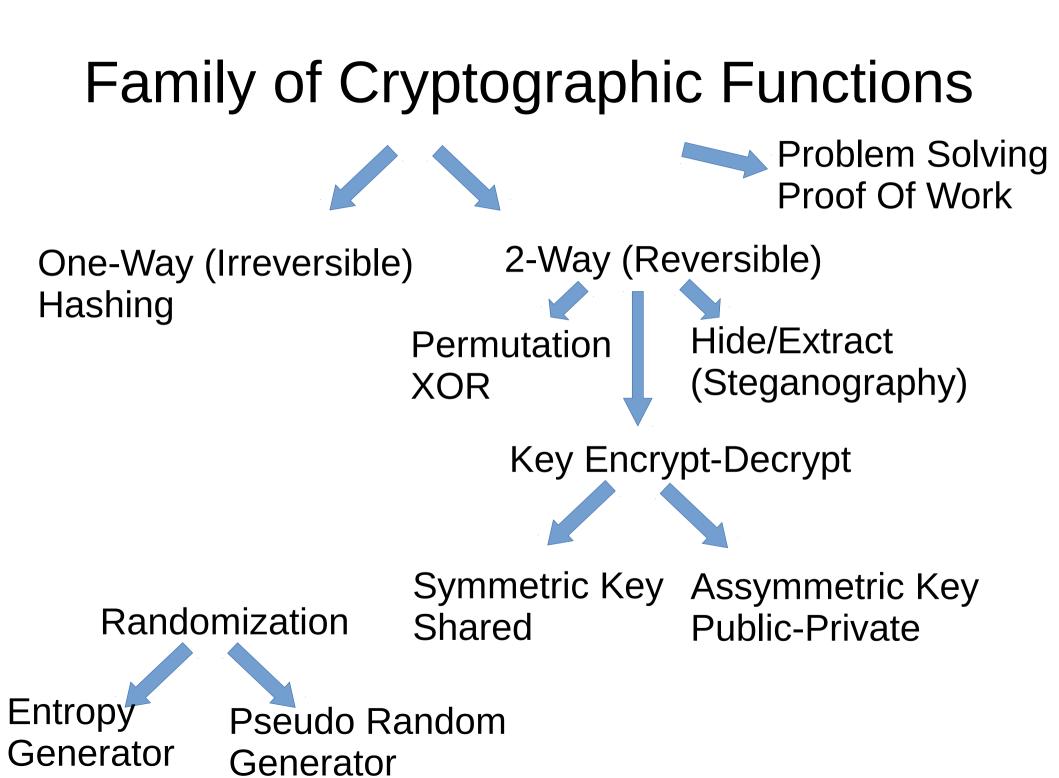


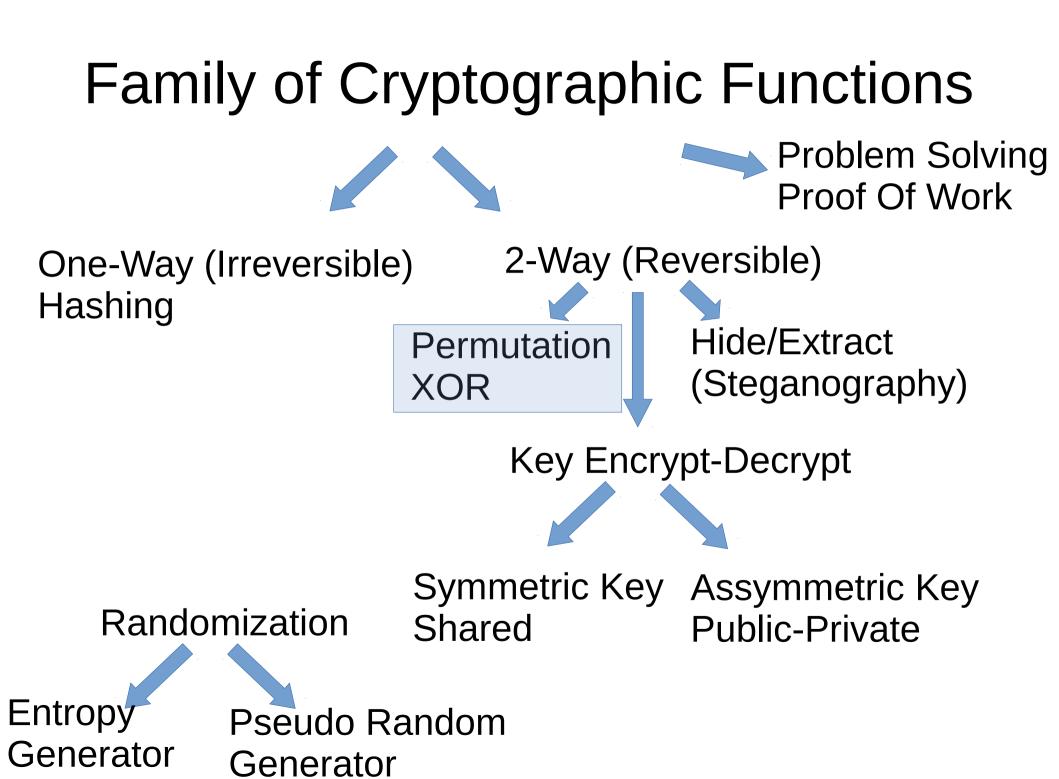
\*\* Switch encrypted (shared key) \*\* ( seed =) G2HA0QEG3HHF5SSG.....



#### Generate new Seed Random Generator



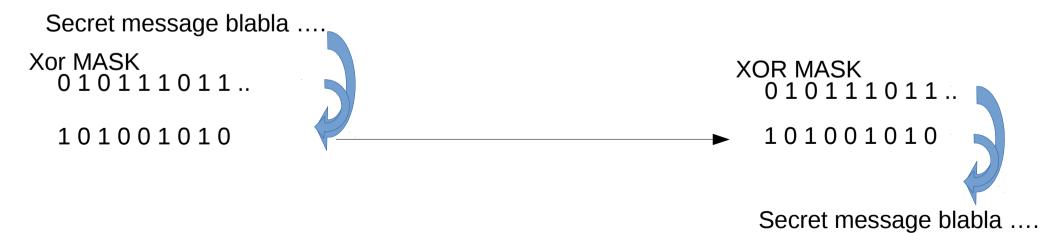






#### VERY VERY Fast

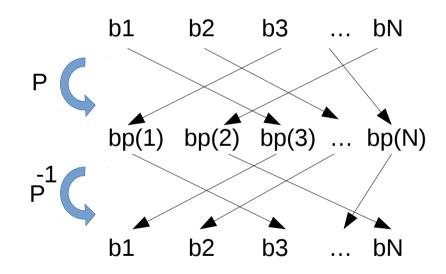
Works Great with "Good" Pseudo Random Generators

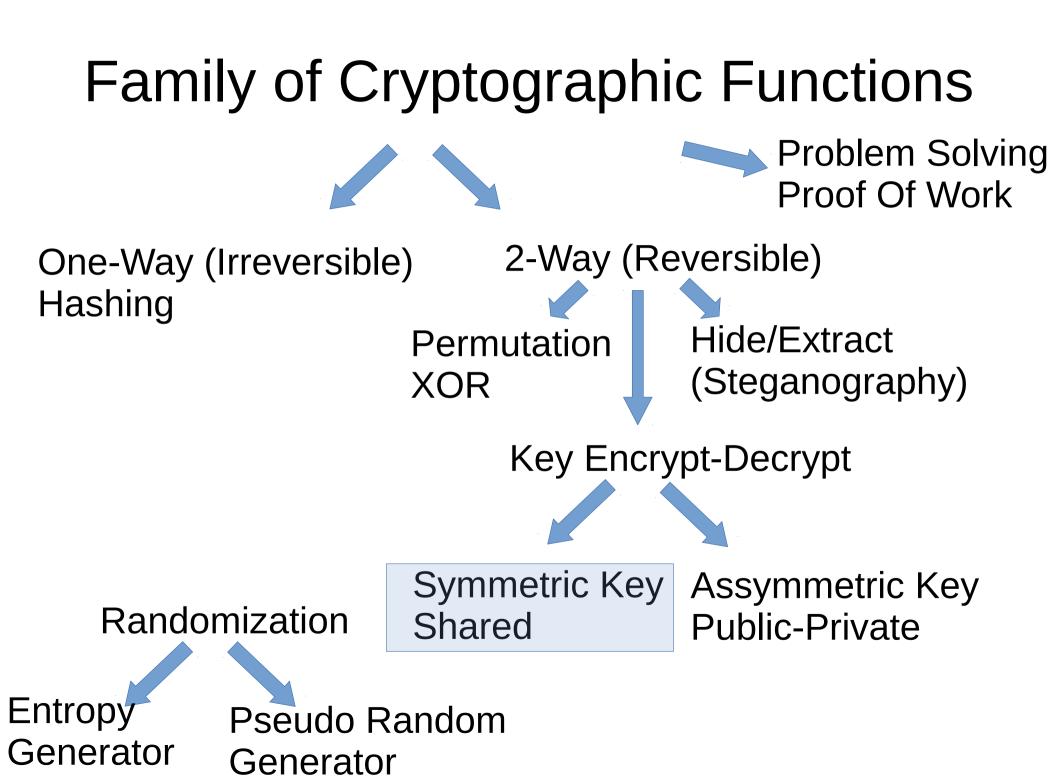


#### Permutations

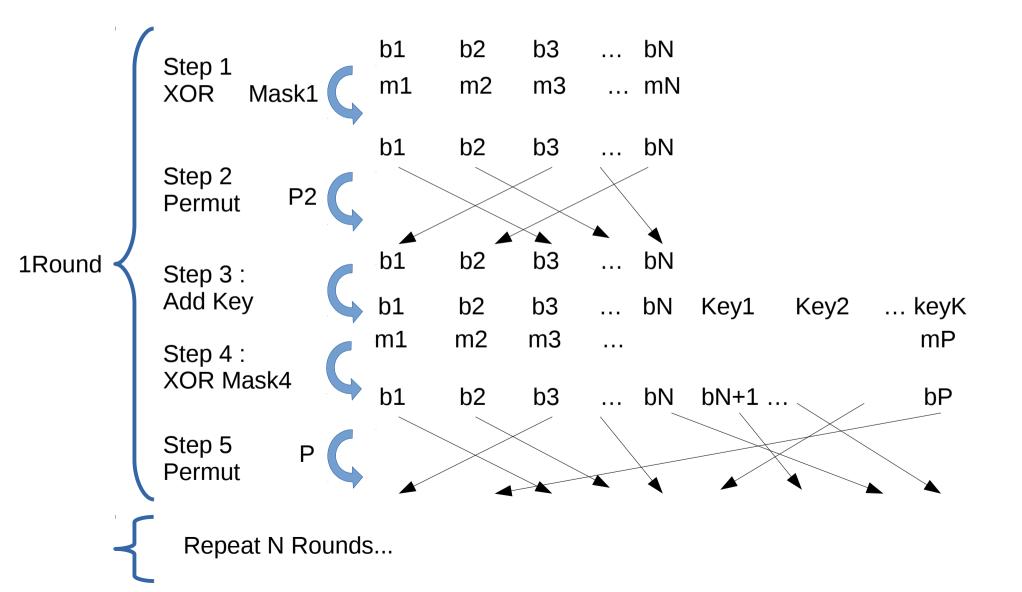
P: [1..N] → [1..N]

P is a permutation if it is inversible (all elts are reached once)





# Combining XOR & Permutations & Add



#### Example : AES, 3-DES, ...

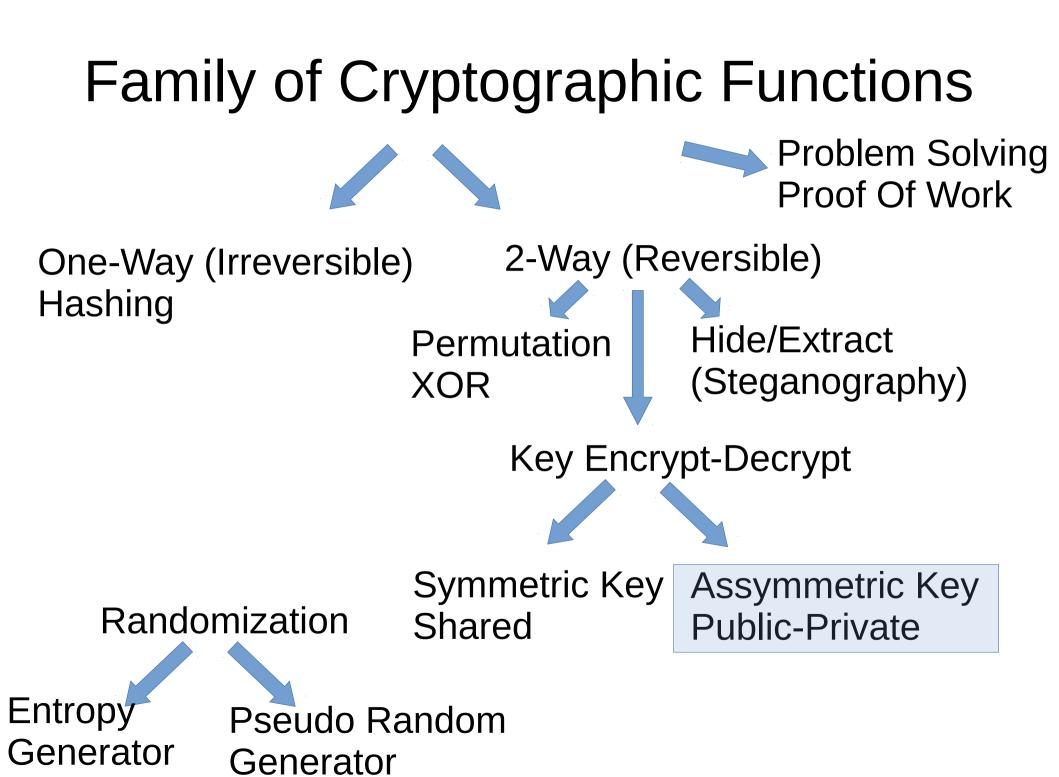
# Article Talk Read Edit View histo

#### Triple DES

From Wikipedia, the free encyclopedia (Redirected from 3-DES)

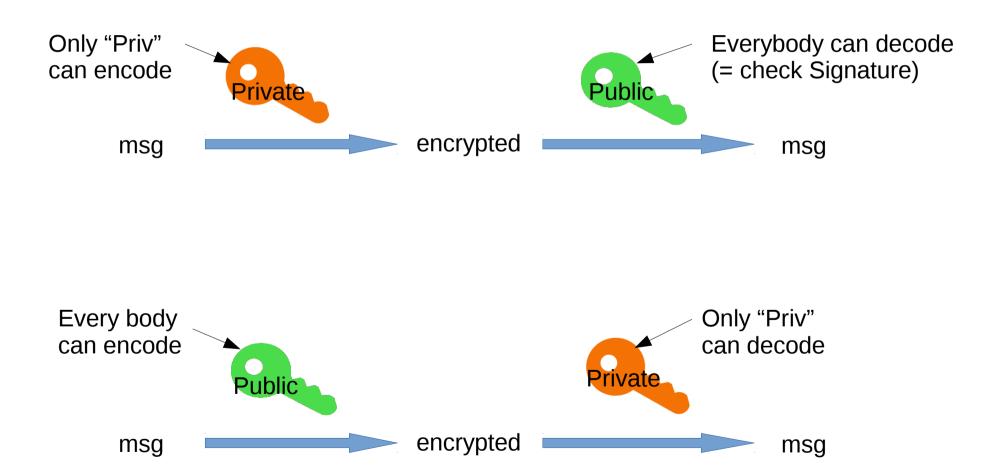
In cryptography, **Triple DES (3DES)**, officially the **Triple Data Encryption Algorithm (TDEA** or **Triple DEA)**, is a symmetric-key block cipher, which applies the Data Encryption Standard (DES) cipher algorithm three times to each data block.

The original DES cipher's key size of 56 bits was generally sufficient when that algorithm was designed, but the availability of increasing computational power made brute-force attacks feasible. Triple DES provides a relatively simple method of increasing the key size of DES to protect against such attacks, without the need to design a completely new block cipher algorithm.

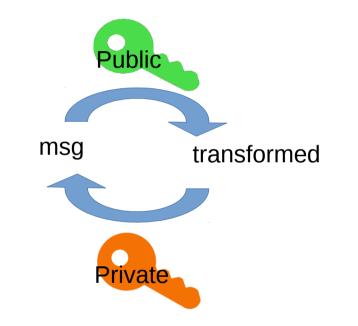






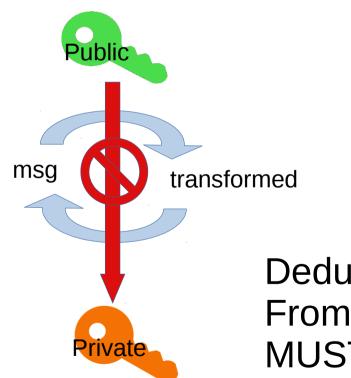


### Public Keys – Private Keys...



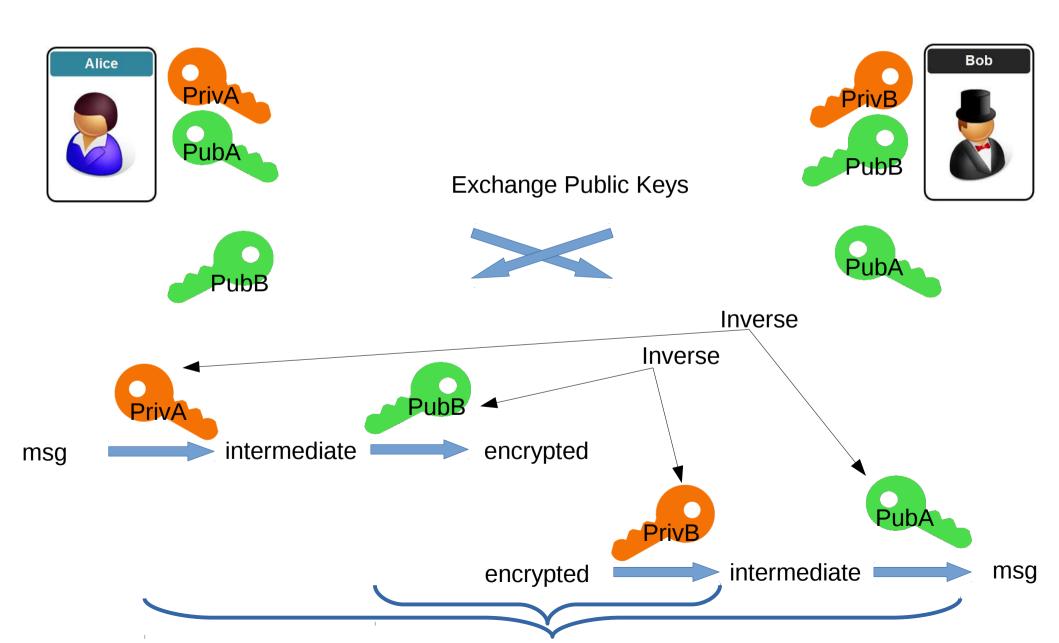
Keys define 2 operations : inverse of each others Like  $+X \rightarrow -X$  $*X \rightarrow /X$ ^N  $\rightarrow$  sqrt^N

### But Deducing Keys MUST Be Impossible/Difficult

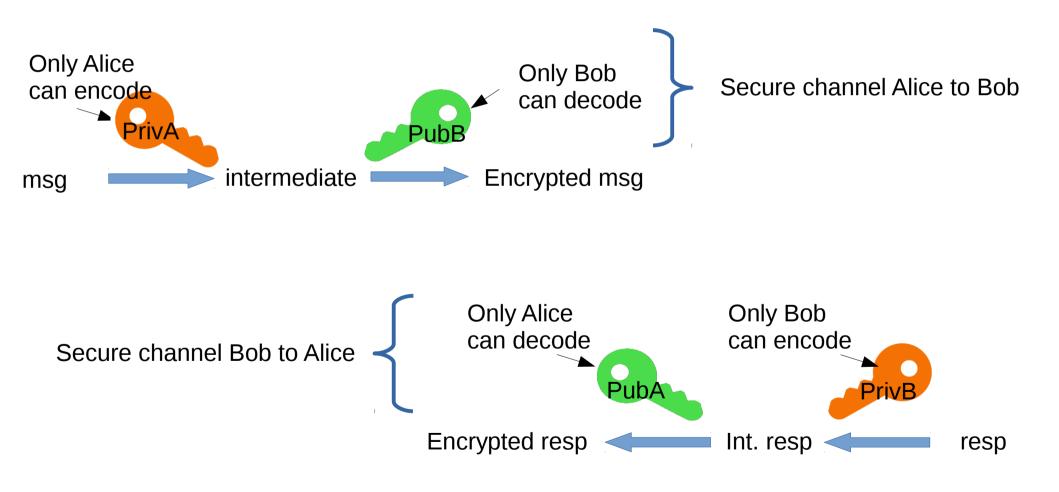


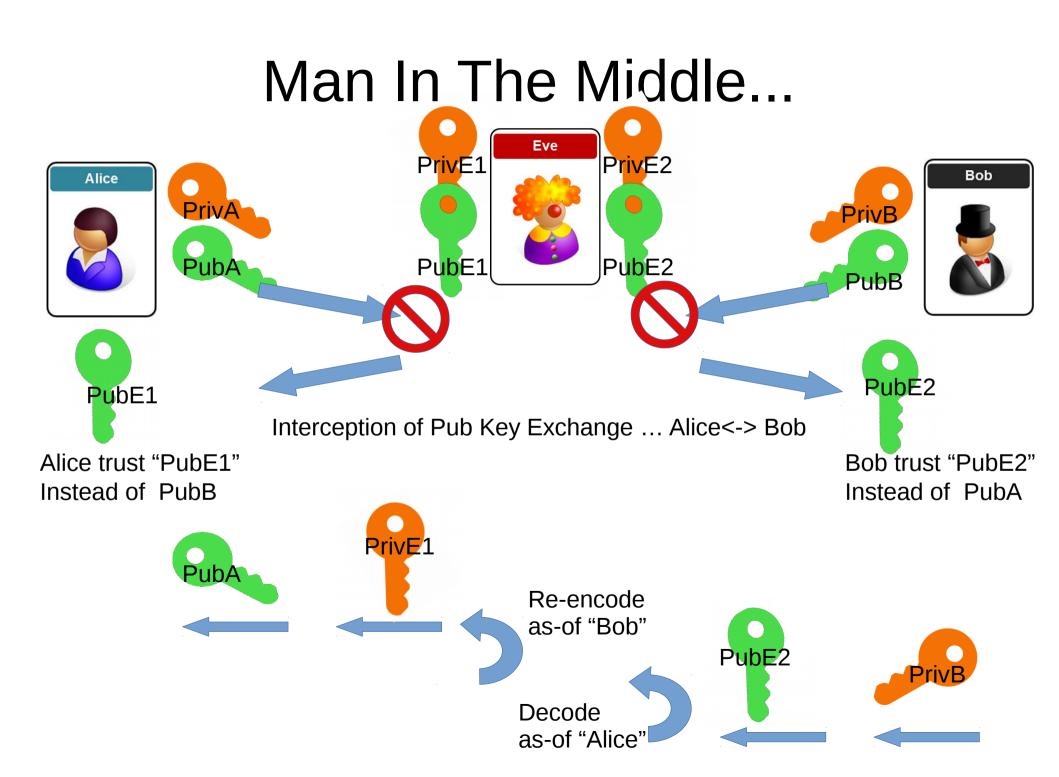
Deducing Private Key From Public Key MUST Be Impossible

### Combining 2 Pairs of Pub-Priv Keys



### Assymetric Keys For Secured 2-Way Channel





### **RSA : Rivest Shamir Adleman**

Read Edit

Article Talk

#### RSA (cryptosystem)

From Wikipedia, the free encyclopedia

For other uses, see RSA (disambiguation).

**RSA** is one of the first practical public-key cryptosystems and is widely used for secure data transmission. In such a cryptosystem, the encryption key is public and differs from the decryption key which is kept secret. In RSA, this asymmetry is based on the practical difficulty of factoring the product of two large prime numbers, the factoring problem. RSA is made of the initial letters of the surnames of Ron Rivest, Adi Shamir, and Leonard Adleman, who first publicly described the algorithm in 1978. Clifford Cocks, an English mathematician working

### **Basic Math: Modulo definition**

Remember Euclidian Division

Let N a number > 0

Every number divided by N has a quotient and a remainder – also called modulo

x / n : for all x => there exist(uniq) q & r with  $0 \le r \le n$ x = q n + r

Modulo N function:  $x \rightarrow x \mod[n] = r$ 

### Algebra Operation on Modulos

 $(x + y) \mod [n] = (x \mod [n] + y \mod [n]) \mod [n]$  $(x * y) \mod [n] = (x \mod [n] * y \mod [n]) \mod [n]$ 

Reasonning on equivalence classes, for equivalence relation "R": a R b <=> a-b mod[n]=0

$$\mathbb{Z}/n\mathbb{Z} = \{\dot{0}, \dot{1}, \dot{2}, \dots \dot{N-1}\} \qquad \overrightarrow{x+y} = \dot{x} + \dot{y}$$
$$\overrightarrow{x\cdot y} = \dot{x} \cdot \dot{y}$$
$$\overrightarrow{x\cdot y} = \dot{x} \cdot \dot{y}$$
$$\overrightarrow{x^{y}} = \dot{x}^{y}$$

### Example on modulo 9

Doing modulo 9 as a 9-year old boy at school ... sum digits to check operations

$$123.456 \mod [9] = 56088 \mod [9] = (5+6+0+8+8) \mod [9] = 0$$
$$= (1+2+3).(4+5+6) \mod [9] = 6.6 \mod [9] = 0$$

This works because 10 mod[9]=1 .. 100 mod[9]=1, 1000 mod[9]=1 so whatever X written in decimal format:

 $(a_n 10^n + a_{n-1} 10^{n-1} + ... a_2 10^2 + a_1 10 + a_0) mod[9] = (a_n + ... + a_1 + a_0) mod[9]$ 

# When p is prime Z/pZ\* is a multiplicative Group

Remember Group definition ?

G = { g0, g1, ...gN } a Set of elements with an operation "." (inside G) Is a "group over ."

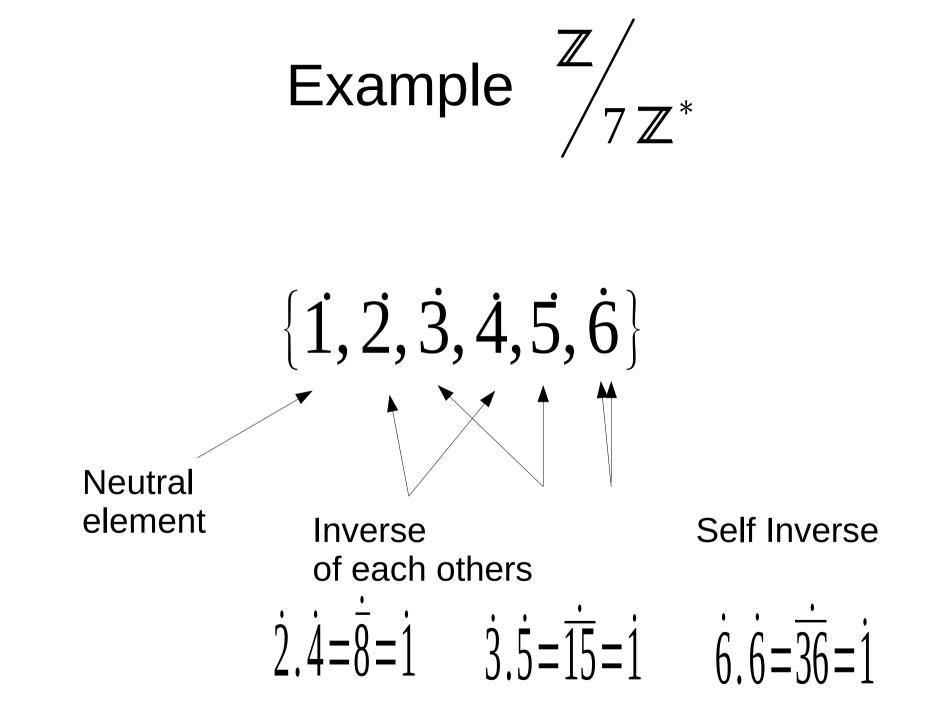
If it exists a neutral element "e"

 $\forall a, e.a=a.e=e$ 

 $\forall a, \exists b | a.b = b.a = e$ 

every element has an inverse:

(b is unique and is the inverse of a :  $b = a^{-1}$ )



### Consecutive Powers ... (=Orbits)

$$a \in \mathbb{Z}/p\mathbb{Z}^*$$
, p prime

$$\{a, a^2, a^3, a^4, \dots a^{p-1}, \dots\}$$
 Is inside finite set..  
so is cyclic ...

let k2 index be the first repetition of already seen k1 elt:

$$a^{k^2} = a^{k^1}$$

Taking inverse

$$a^{k_2-k_1}=e$$

Proof next... that k2-k2=p-1

### Orbits are all similar... all equals

whatever a, the orbit contains e... so orbits contains

$$\{a, a^{2}, .., a^{p-2}, e\}$$

For 2 elements a & b ... orbits of a and b are identical, "scaled" By a factor  $a^{-1}b$ 

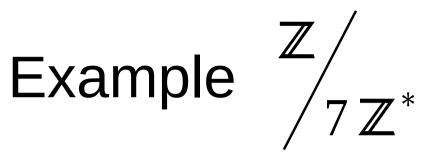
So For any orbit, card Orbit = cst

If they are N orbits, they all cover the set  $\mathbb{Z}_{p\mathbb{Z}^*}$ 

If p is a prime number ... (sub-orbit N can not divide P prime)

а

... There is only 1 orbit, containing ALL elements !



Start for example with a = 3Compute  $a, a^2 \dots$ 

$$a = \dot{3}$$

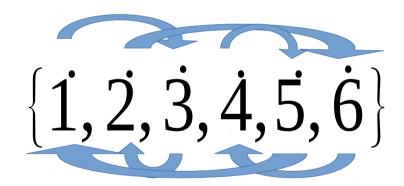
$$a^{2} = \dot{3}\dot{3} = \dot{9} = \dot{2}$$

$$a^{3} = \overline{3.2} = \dot{6}$$

$$a^{4} = \overline{3.6} = \dot{4}$$

$$a^{5} = \overline{3.4} = \dot{5}$$

$$a^{6} = \overline{3.4} = 5$$



### "Small Theorem" of Fermat

 $\forall a \in \mathbb{Z}/p\mathbb{Z}^*$ , p prime

$$a^{p-1} = 1 \mod[p]$$

$$a^{p} = a \mod [p]$$

### Relationship with Crypto? ...

For p1,p2 2 prime numbers...

$$a^{p1} = a \mod [p1]$$
  
 $a^{p2} = a \mod [p2]$ 

$$a^{p1.p2} = (a^{p1})^{p2} = a \mod [p1p2]$$
  
encrypt decrypt

Intuitively ... Try decompose p1.p2 as a different product of 2 key parts "e.d mod[..]" ... but not as p1.p2 (too easy) !

Technically.. solve e and d...  
$$e \cdot d = 1 \mod[\varphi(p 1 p 2)]$$

Then  $(a^e)^d = a \mod [p \ 1 \ p \ 2]$ And also  $(a^d)^e = a \mod [p \ 1 \ p \ 2]$ 

### Inverting Key .. need Decomposing N in p1.p2

Choose 2 HUGE primes p1,p2

Multiply N=p1.p2

... give N to someone

and reward 1M\$ IF he finds back p1 & p2

### Naive Search...

Given N Try decompose :

```
for ( BigInteger i = N; ; i = i-2 ) {
    if (divides(i, N)) {
        p1 = i; break;
    }
}
```

### How HUGE is HUGE enough?

Example: choose  $p1 > 2^{1024}$ =2.2.2. .... (1024 times) ~ 1000 . 1000 .... (1014 times) ~ 1 000 000 000 .... (3000 zeros)

In Theory : Decomposition Program finishes

In Practise :

- More than Atoms in Universe
- Slow... 1CPU => 1000...00000 centuries
- Energy : 1000...000 CPUS => requires more power than 1000...000 solars

### Possible to Find so HUGE Primes?

In Theory : infinite numbers of Primes

Storing all primes from [0, 2^1024] ?? ... is even harder than counting them

Primes become rare : Count primes in [0, N] ~ log N

There always exist a prime in [k, 2k]

### Find One Random (!) Huge Prime Efficiently (?)

Given start = random number > 2^N-1 ... with N~1024

```
for ( BigInteger i = start; ; i = i+2 ) {
    if (isPrime(i)) {
        p = i; break;
    }
}
```

## Need Efficient Primality Test $p is prime \Leftrightarrow$ $\forall a \in \mathbb{Z}_{p \mathbb{Z}^*} \quad a^{p-1} = 1 \mod[p]$

Choose any a, for example a=27 Compute  $a^{p-1}[p]$ 

If != 1p is NOT primeIf == 1Probability 1/p tl

Probability 1/p that p is anyway not prime

Reapeat for b=43 Confirm .. or proba 1/p^2

### Efficient Compute Powers..

Write in basis-2 (binary representation)

$$p-1=b_0 2^0+b_1 2^1+...b_n 2^n \quad \forall i,b_i \in \{0,1\}$$

Then

$$a^{p-1} = a^{b_0} \cdot a^{2b_1} \dots a^{2^n b_n}$$
  
=  $\{a^1 if b_1 = 1 \\ 1 if b_1 = 0\} \cdot \{a^2 if b_2 = 1 \\ 1 if b_2 = 0\} \dots \{a^{2^n} if b_n = 1 \\ 1 if b_n = 0\}$   
square square

### Efficient (Enough?) Powers Compute

#### $a^{p-1}$ Computable in o(2.N multiplications) BUT Would be a HUGE number ...

#### $a^{p-1}[p]$ Computable in o(2.N multiplications + 1.N modulo) ... quite FAST, with small memory need

### Not Efficient Enough For Video & High Traffic Network !!!

But OK for small DATA

~ 1ms on a CPU

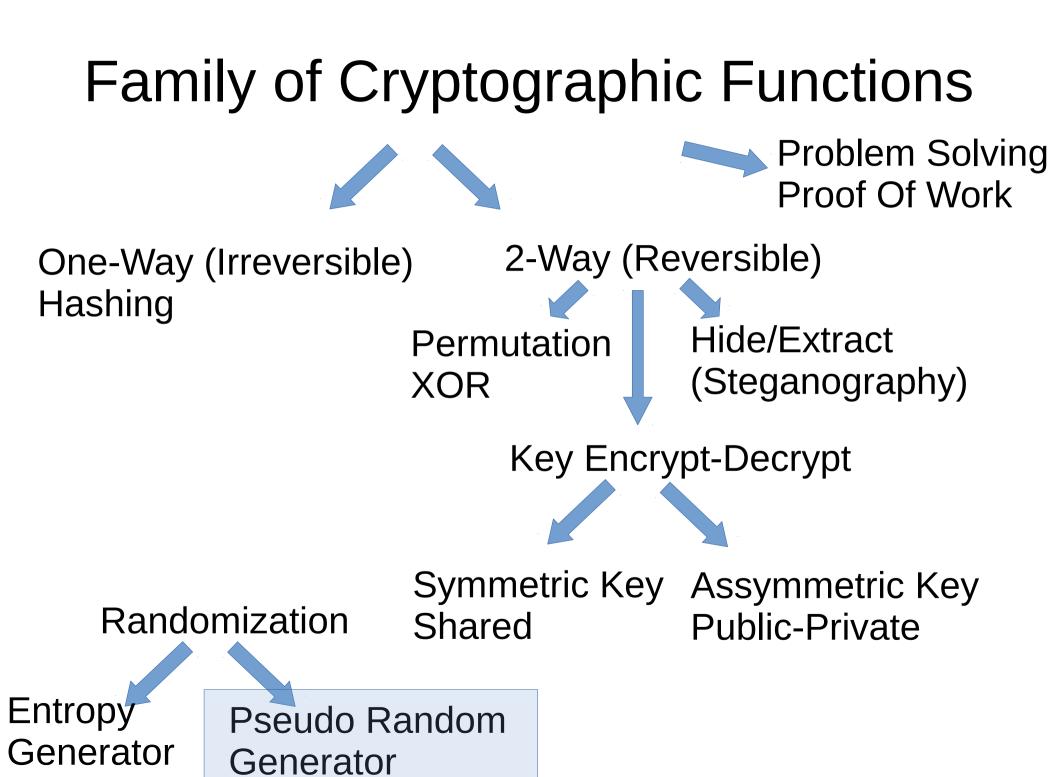
Embeddable in RDA SecurID card ship



**OK For Exchanging a Seed** Pseudo Generator ... then using XOR encoding

Application : **TLS** protocol (ex SSL), for example in





### Fast Pseudo-Random Generators

Back to Math : Doing Polynomials

$$P[X] = a_0 + a_1 X + a_2 X^2 + ... a_n X^n$$

... Polynomials over Z/2Z modulos, truncated to Rank N

$$X \in \frac{Z}{2Z} = \{0, 1\} \quad \forall i, a_i X \in \frac{Z}{2Z} \quad \Rightarrow P[X] \in \frac{Z}{2Z}$$

... we are interrested not in X ... put in P (in coefficients) algebra of P : P1, P2, ... P1.P2, P1+P2

### **Consecutive Powers of P**

Consecutive (truncated) powers of P :

$$\{P[X], P^{2}[X], P^{3}[X]...P^{2^{N}}[X]...P^{2^{N}}[X]...\}$$

If "P" is chosen irreductible in Z/2Z[n] ... then orbit contains all polynoms! (all coefficients of all polynoms)

$$card(\{a_1 a_2 \dots a_n\})=2^{2^n}$$

Suppose N = 64 ... then  $2^{2^{64}} \simeq 2^{10^9}$  .. a Pseudo-Random generator HUGE cycle

### Multiply (and Truncate) Polynoms

$$A[X] = a_0 + a_1 X + a_2 X^2 + ... a_n X^n$$
$$B[X] = b_0 + b_1 X + b_2 X^2 + ... b_n X^n$$

 $A \cdot B[X] = A[X] \cdot B[X] = a_0 \cdot b_0 + (a_0 b_1 + a_1 b_0) X$ +  $(a_0 b_2 + a_1 b_1 + a_2 b_0) X^2 + .. (a_0 b_n + .. a_n b_0) X^n$ + truncated ..  $X^{n+1} \cdot .. X^{2n}$ 

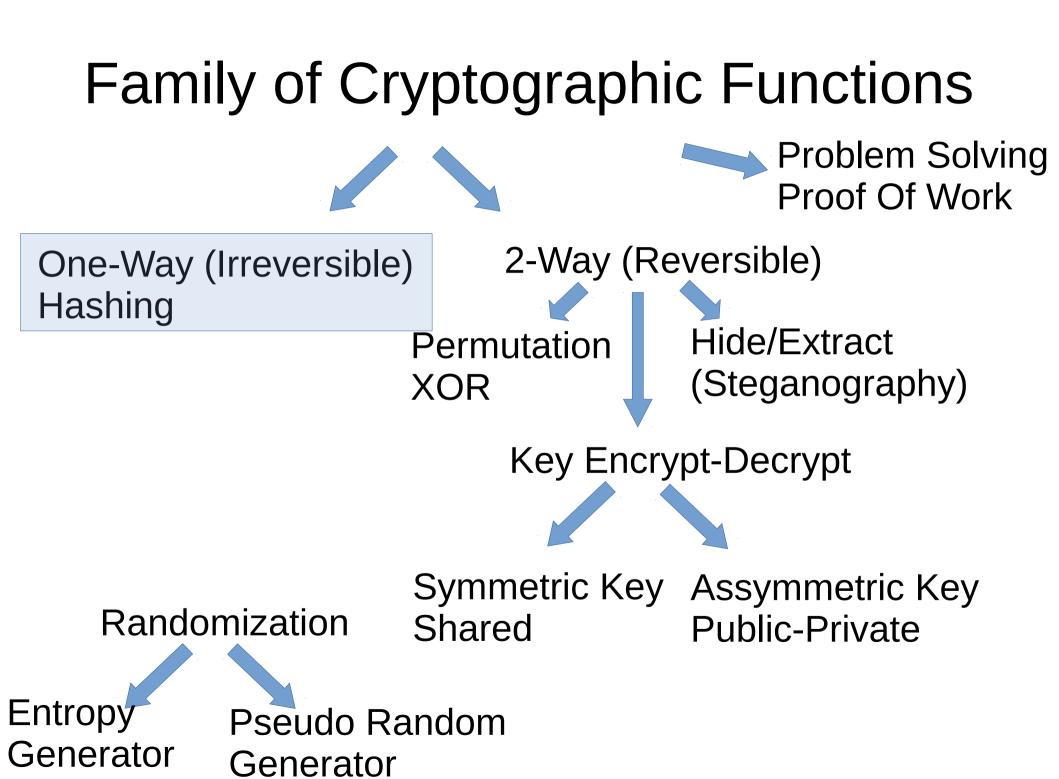
### Efficient for P=1+X using Bit Shift, Bit And..

 $A[X] = a_0 + a_1 X + a_2 X^2 + ... a_n X^n$ B[X] = 1 + X

 $A.B[X] = a_0 + (a_0 + a_1) X + (a_1 + a_2) X^2 + \dots$ 

$$a_{1} a_{2} a_{3} \dots a_{n-1} a_{n}$$
+
$$a_{2} a_{3} a_{4} \dots a_{n} 0 = Bit Shift$$

$$(a_{1}+a_{2}) (a_{2}+a_{3}) \dots (a_{n-1}+a_{n}) a_{n}$$

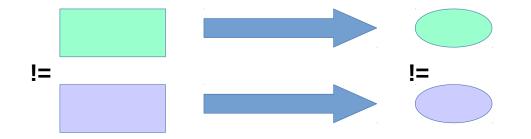


### Hashing ...



## Hashing is One-Way transformation

Hashing different product usually give different result



# Hashing ... must be equi-distributed minimise "Collisions"

A Hashing Collision



Of course if can happen ... example in java: "public int hashCode()"

by default Object.hashCode = System.identityHashCode(x)

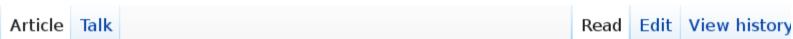
- = ... 64bits pointer (when first hashed)  $\rightarrow$  hashed to 32 bits
- = (hashPtr64 ^ (hashPtr64 >>>32))

### Hashing : MD5, SHA-1, SHA-256, SHA-512 ..

Article Talk

Read Edit View h

#### MD5



From Wikipedia, the fre (Redirected from Md5

The MD5 algorithn hash value. Although cryptographic hash fu vulnerabilities. It can but only against unir Like most hash funct cracked by brute-forc detailed in the securi

MD5 was designed b function MD4.<sup>[3]</sup> The SHA-1

From Wikipedia, the free encyclopedia

In cryptography, SHA-1 (Secure Hash Algorithm 1) is a cryptographic hash function designed by the United States National Security Agency and is a U.S. Federal Information Processing Standard published by the United States NIST.<sup>[3]</sup> SHA-1 produces a 160-bit (20-byte) hash value known as a message digest. A SHA-1 hash value is typically rendered as a hexadecimal number, 40 digits long.

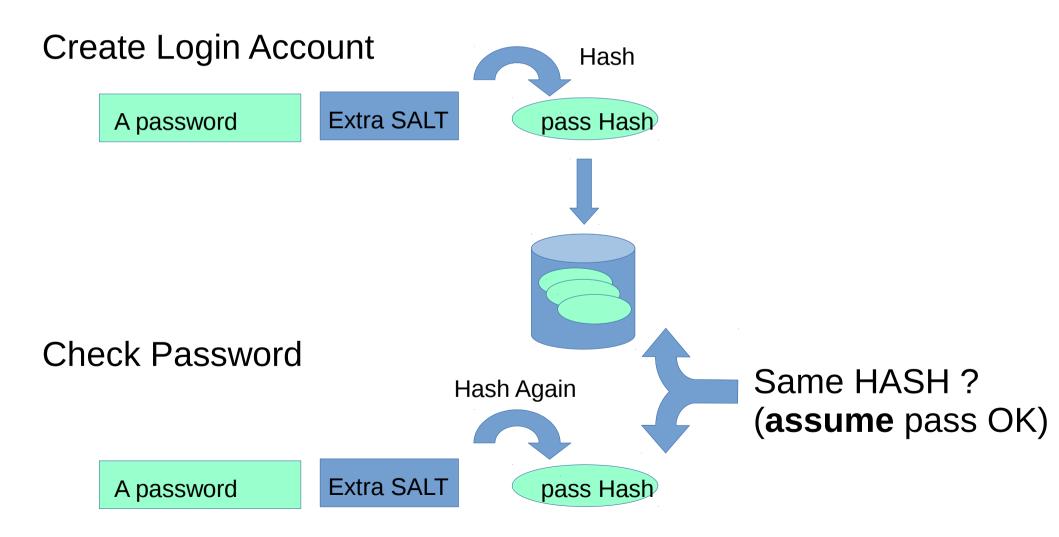
SHA-1 is no longer considered secure against well-funded opponents. In 2005, cryptanalysts found attacks on SHA-1 suggesting that the algorithm might not be secure enough for ongoing use,<sup>[4]</sup> and since 2010 many organizations have RSA license. The abbreviation "MD" stands for "Message Digest."

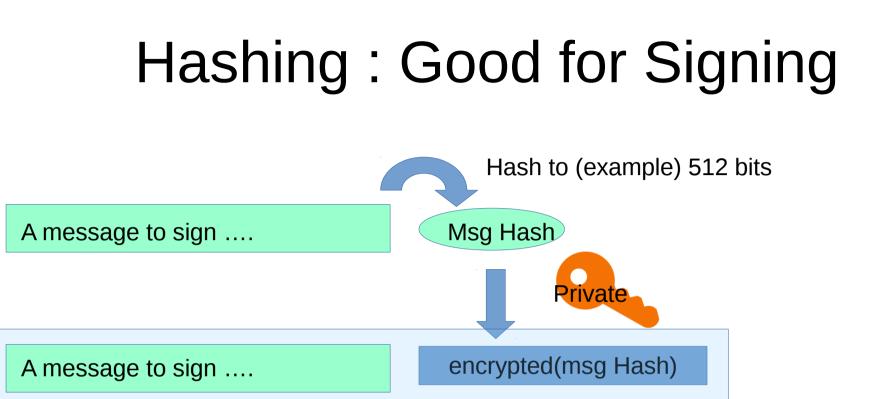
The security of the MD5 has been severely compromised, with its

weaknesses having been exploited in the field, most infamously by the

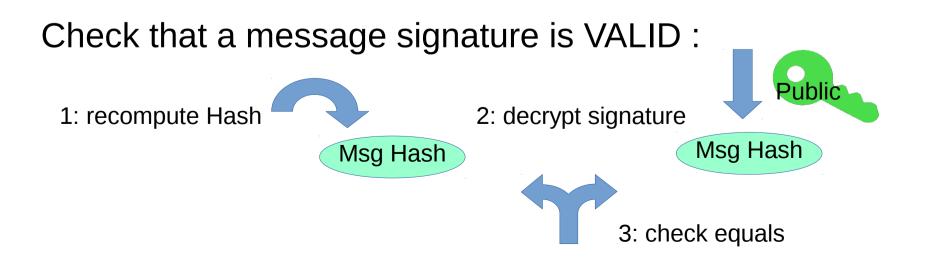
Flame malware in 2012. The CMU Software Engineering Institute considers

### Hashing For Storing "Passwords"

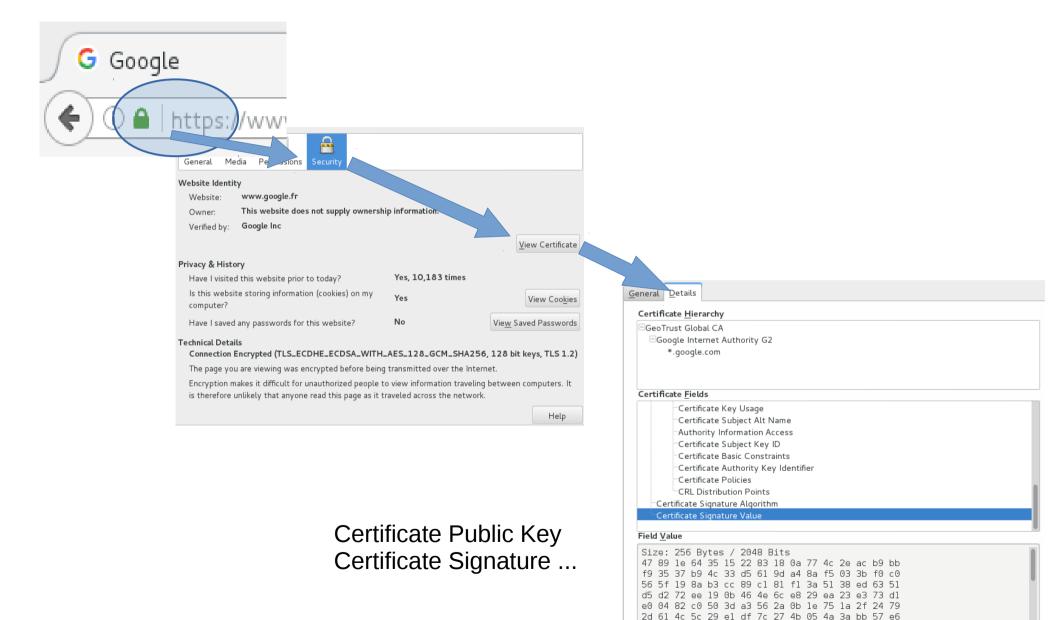




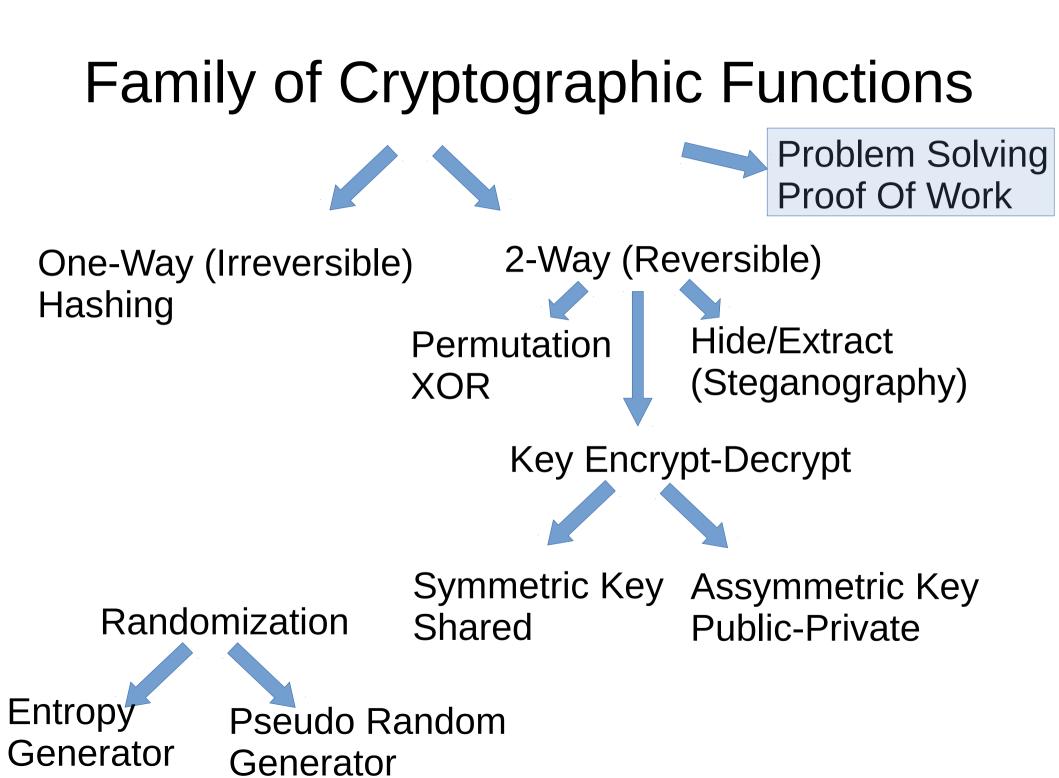
Signed Message : clear text + extra signature (at bottom of page)



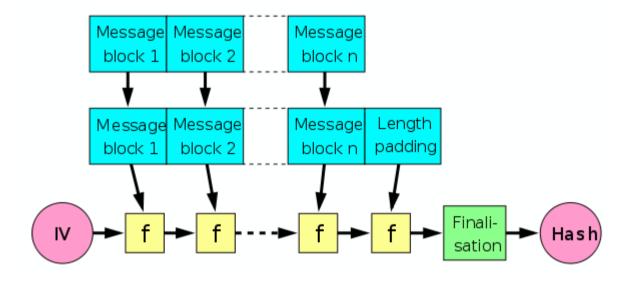
### Certificates = Signed Public Keys



db 24 30 3b 52 b8 b1 8a e6 22 bd 64 22 c1 64 35



### Hash a BitCoin Transactions Chain BlockChain



### Merkle Tree

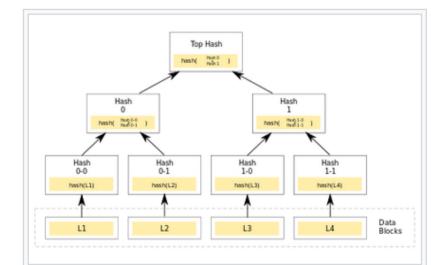
Article	Talk	Read	Edit	View history	Search Wikipedia	Q
M	erkle tree					<b>(())</b>

From Wikipedia, the free encyclopedia

In cryptography and computer science, a hash tree or Merkle tree is a tree in which every non-leaf node is labelled with the hash of the labels or values (in case of leaves) of its child nodes. Hash trees allow efficient and secure verification of the contents of large data structures. Hash trees are a generalization of hash lists and hash chains.

Demonstrating that a leaf node is a part of the given hash tree requires processing an amount of data proportional to the logarithm of the number of nodes of the tree;<sup>[1]</sup> this contrasts with hash lists, where the amount is proportional to the number of nodes.

The concept of hash trees is named after Ralph Merkle who



An example of a binary hash tree. Hashes 0-0 <sup>□</sup> and 0-1 are the hash values of data blocks L1 and L2, respectively, and hash 0 is the hash of the concatenation of hashes 0-0 and 0-1.

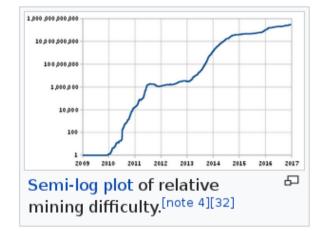
### Proof Of Work, Mining

#### Mining [edit]

*Mining* is a record-keeping service.<sup>[note 5]</sup> Miners keep the blockchain consistent, complete, and unalterable by repeatedly verifying and collecting newly broadcast transactions into a new group of transactions called a *block*.<sup>[29]</sup> Each block contains a cryptographic hash of the previous block,<sup>[29]</sup> using the SHA-256 hashing algorithm,<sup>[8]:ch. 7</sup> which links it to the previous block,<sup>[29]</sup> thus giving the blockchain its name.

In order to be accepted by the rest of the network, a new block must contain a so-called *proof-of-work*.<sup>[29]</sup> The proof-of-work requires miners to find a number

called a nonce, such that when the block content is hashed along with the



nonce, the result is numerically smaller than the network's *difficulty target*.<sup>[8]:ch. 8</sup> This proof is easy for any node in the network to verify, but extremely time-consuming to generate, as for a secure cryptographic hash, miners must try many different nonce values (usually the sequence of tested values is 0, 1, 2, 3, ...<sup>[8]:ch. 8</sup>) before meeting the difficulty target.

### Other BlockChains (for contracts, legal ownership, assurance, ..)

Article Talk

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#### Ethereum

From Wikipedia, the free encyclopedia

Ethereum is an open-source, public, blockchain-based distributed computing platform featuring smart contract (scripting) functionality, which facilitates online contractual agreements.<sup>[2]</sup> It provides a decentralized Turing-complete virtual machine, the Ethereum Virtual Machine (EVM), which can execute scripts using an international network of public nodes. Ethereum also provides a cryptocurrency token called "ether", which can be transferred between accounts and used to compensate participant nodes for computations performed. Gas, an internal transaction pricing mechanism, is used to mitigate spam and allocate resources on the network.<sup>[2][3]</sup>

### Conclusion

