

arnaud.nauwynck@gmail.com

Introduction to Image Analysis

This document: <http://arnaud-nauwynck.github.io/>

Outline

Linear Filter

Non-Linear Filter : Mathematical Morphology
Erode/Dilate, Open/Close

Visual Cortex – bio mimetism
Illusory shapes

Object Recognition, Segmentation, Optical Flow, ..

Deep Neural Networks

Who's She?

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Lenna

From Wikipedia, the free encyclopedia

This article is about the test image. For other uses, see [Lenna \(disambiguation\)](#).

Lenna or **Lena** is the name given to a [standard test image](#) widely used in the field of image processing since 1973.^[1] It is a picture of [Lena Söderberg](#), shot by photographer [Dwight Hooker](#), cropped from the [centerfold](#) of the November 1972 issue of *Playboy* magazine.

The spelling "Lenna" comes from the [anglicisation](#) used in the original *Playboy* article.

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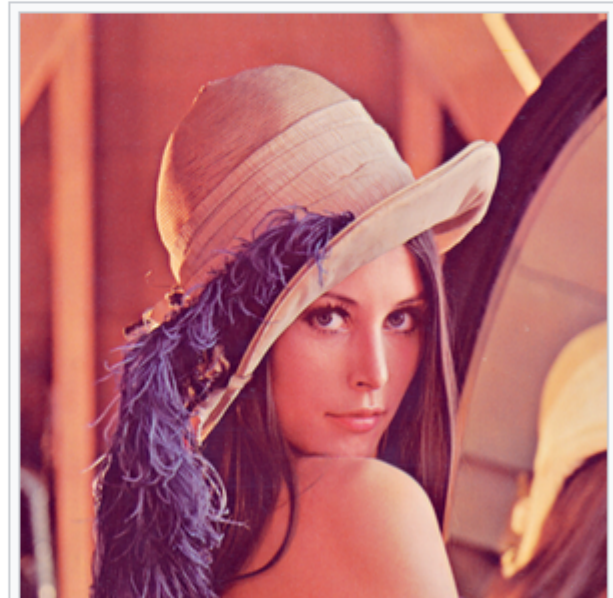


Image of [Lena Söderberg](#) used in many image processing experiments. [\(Click on the image to access the actual 512x512px standard test version.\)](#)

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Linear Combination of Filters

Definition of Linear:

$$\forall I_1, I_2 \text{ image}, \forall a, b \in \mathbb{R}, F(a I_1 + b I_2) = a F(I_1) + b F(I_2)$$

Filter is determined by images of 1 pixel i, j $\delta_{ij}(x, y) = \begin{cases} 1 & \text{if } x=i \wedge y=j \\ 0 & \text{otherwise} \end{cases}$

$$F(\text{Img Zero except at } ij) = F(\delta_{ij}) = F_{ij}$$

Indeed, decomposing $\text{Img} = \sum_{ij} \text{Pixel}_{ij} \delta_{ij}$

$$F(\text{Img}) = F\left(\sum_{ij} \text{img}_{ij} \delta_{ij}\right) = \sum_{ij} \text{img}_{ij} F(\delta_{ij}) = \sum_{ij} \text{img}_{ij} F_{ij}$$

$$\forall x, y F(\text{Img})(x, y) = \sum_{ij} \text{img}_{ij} F_{ij}(x, y)$$

There are N^2 F_{ij} matrix ... and each F_{ij} is a N^2 matrix image

Invariant by Translation

Filter is invariant by translation
when coefficients $F_{ij}(x,y)$ do NOT depend of ij

$$\forall i, j F_{ij} = K = \text{Matr} . N \times N$$

K is called the Kernel of the linear transformation
it is the coefficient applied uniformly (by translation) to all points

This is possible except on borders of image
(translate Kernel reaches outside of image)

What to do at the Boundary ?

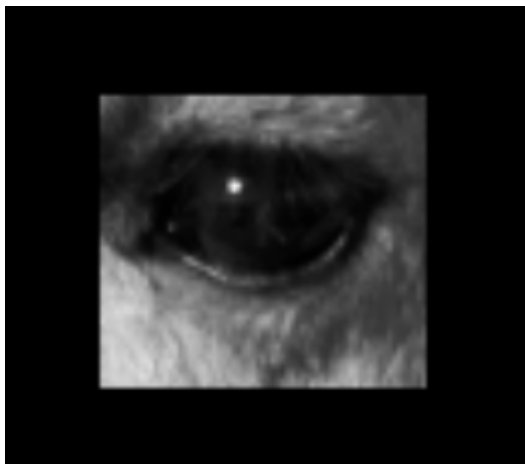
Crop ?



Extend ?



Pad 0 ?



Wrap ?

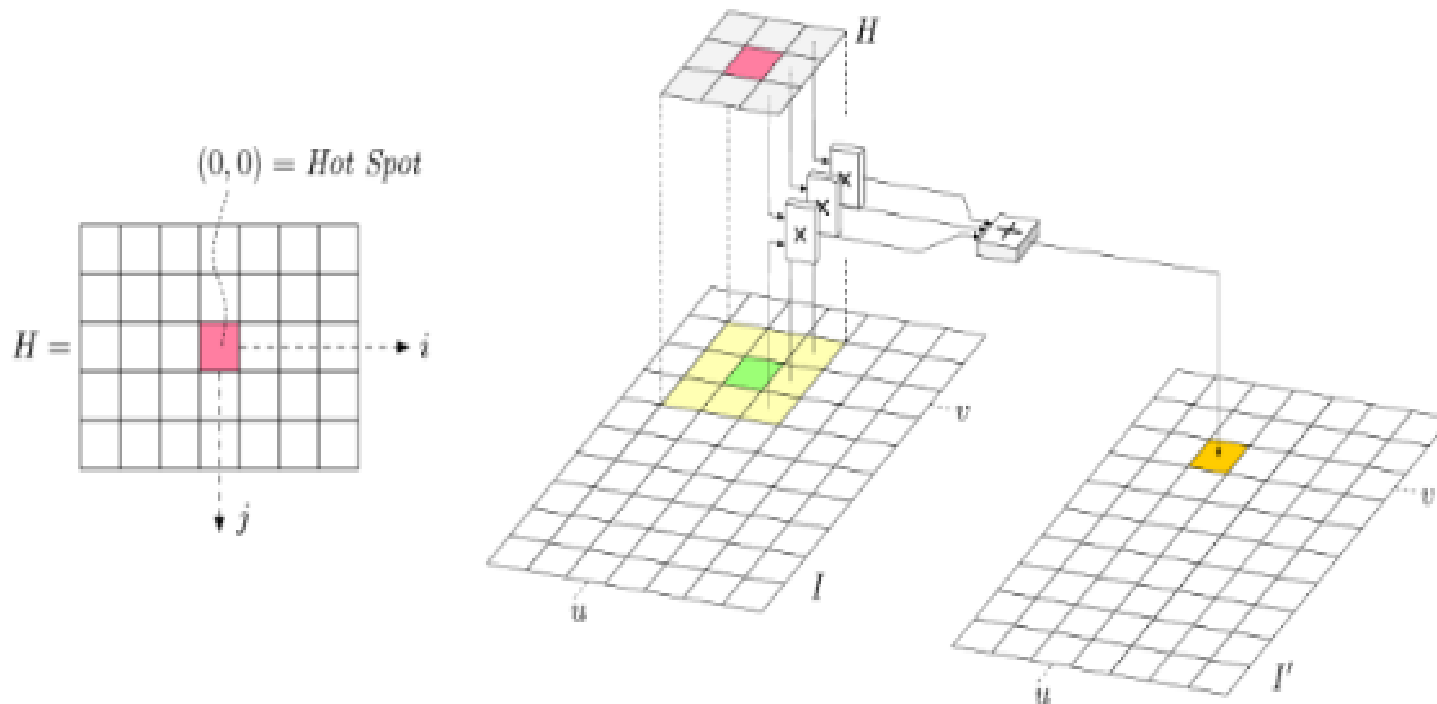


Kernel with small neighborhood only

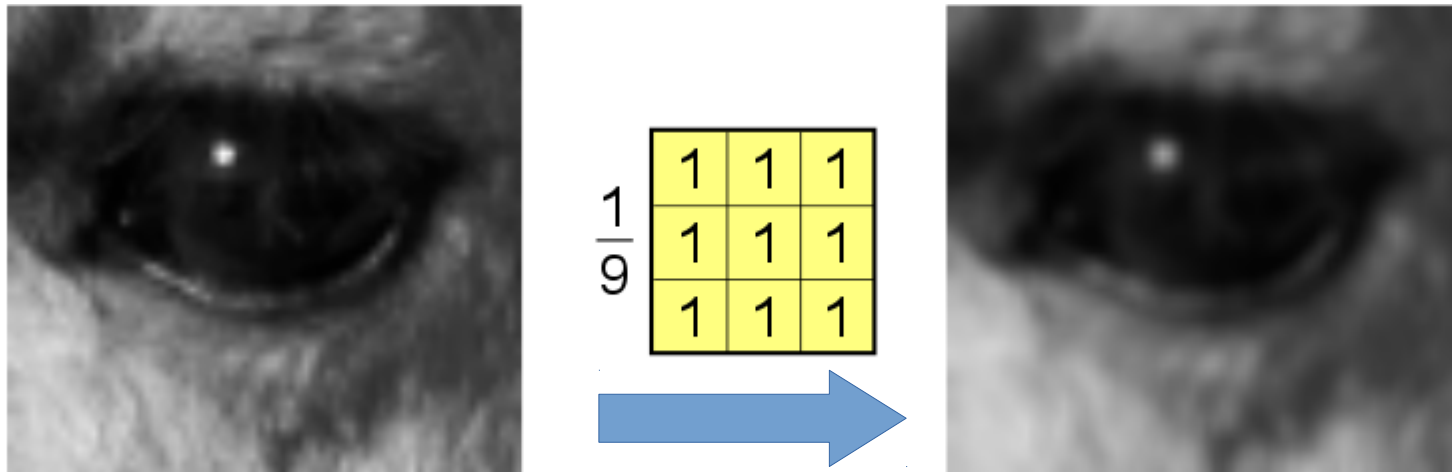
Example : Kernel with 1 neighbor = matrix 1x1 $K = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{311} & k_{32} & k_{33} \end{pmatrix}$

Kernel with 2 neighbors = matrix 5x5

$$K = \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} \end{pmatrix}$$



Example : Average Kernel



For all pixel = take average of all neighbours

Smooth every value

Make border fuzzy

Remove noise of single isolated pixel

2D Kernel Filter = Correlation

Notice that the kernel H is just a small image!

Let $H : R_H \rightarrow [0, K - 1]$

$$I'(u, v) = \sum_{(i,j) \in R_H} I(u + i, v + j) \cdot H(i, j)$$

This is known as a **correlation** of I and H

Correlation ~ Convolution (Symmetric Image)

Definition

Convolution of an image I by a kernel H is given by

$$I'(u, v) = \sum_{(i,j) \in R_H} I(u - i, v - j) \cdot H(i, j)$$

This is denoted: $I' = I * H$

- ▶ Notice this is the same as correlation with H , but with negative signs on the I indices
- ▶ Equivalent to vertical and horizontal flipping of H :

$$I'(u, v) = \sum_{(-i,-j) \in R_H} I(u + i, v + j) \cdot H(-i, -j)$$

Comparison with 1D Time Filter

$$y(t) = \int_0^T x(t - \tau) h(\tau) d\tau$$

$$y_k = \sum_{i=0}^N x_{k-i} h_i$$

Properties of Convolutions ...

$$\text{Img} * K = K * \text{Img}$$

we can leave the image fixed and slide the kernel or leave the kernel fixed and slide the image.

$$\text{Img} * (aK_1 + bK_2) = a \text{Img} * K_1 + b \text{Img} * K_2$$

Combine linear convolutions..

$$(\text{Img} * K_1) * K_2 = \text{Img} * (K_1 * K_2)$$

Instead of applying K_1 to Img then K_2 to result,
We can compute $(K_1 * K_2)$ and apply it to Img

Properties of Separation x/y

$$H_x = [1 \ 1 \ 1 \ 1 \ 1], \quad \text{and} \quad H_y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$H = H_x * H_y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Instead of applying H ... it is faster applying H_x then H_y

Repeated Smoothing Effect of Filter Size

Repeat 7 times 3×3 Kernel = 3 times 7×7 Kernel ...

Mean Filters:



Original



7×7



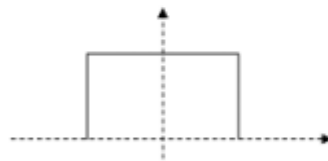
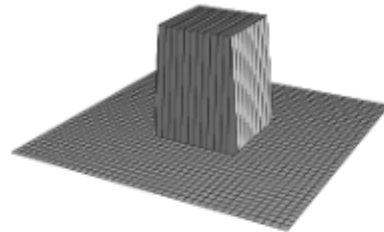
15×15



41×41

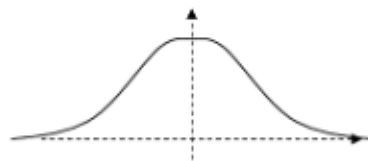
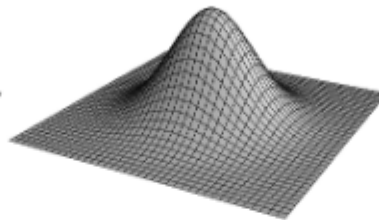
Box, Gaussian, Laplacian (=Gaussian Derivative 1), ...

Box



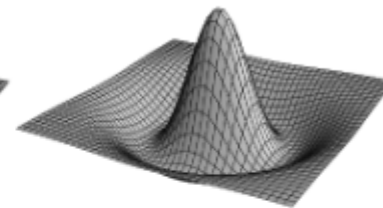
0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Gaussian



0	1	2	1	0
1	3	5	3	1
2	5	9	5	2
1	3	5	3	1
0	1	2	1	0

Laplace



0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

Gaussian..

In 1D:

$$g_{\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

In 2D:

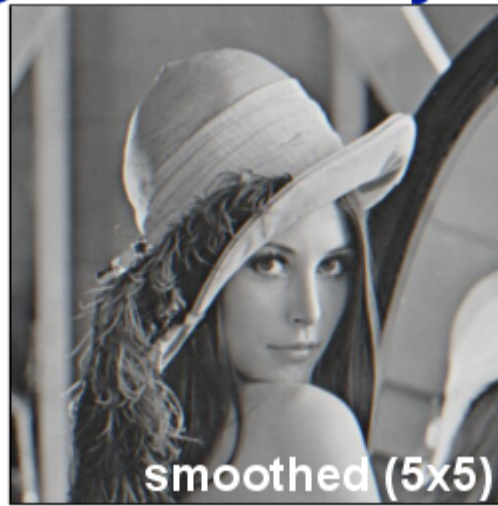
$$\begin{aligned} G_{\sigma}(x, y) &= \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \\ &= g_{\sigma}(x) \cdot g_{\sigma}(y) \end{aligned}$$

(A 2-D Gaussian Filter is the product of 2 1-D gaussian)

Blurr = Identity - Smooth



-



=



+ a



=



Gradient = Derivatives...

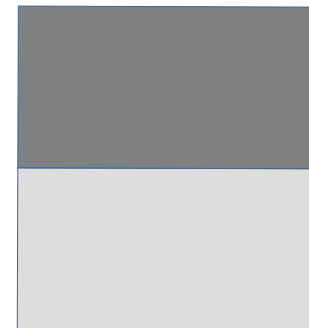
→ $Gradient_x = \begin{pmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{pmatrix}$

Correlation = Detect changes



↓ $Gradient_y = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{pmatrix}$

Correlation = Detect changes



GradX .. GradY

GradX

=> detect Vertical Edges



GradY

=> detect Horizontal Edges

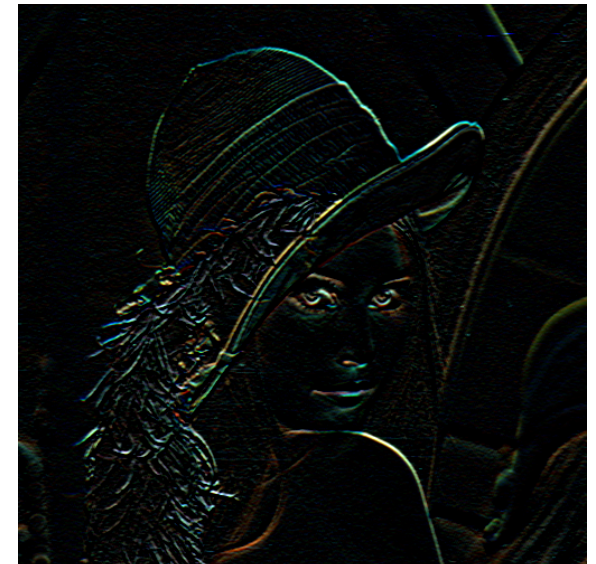


Look Closely ..




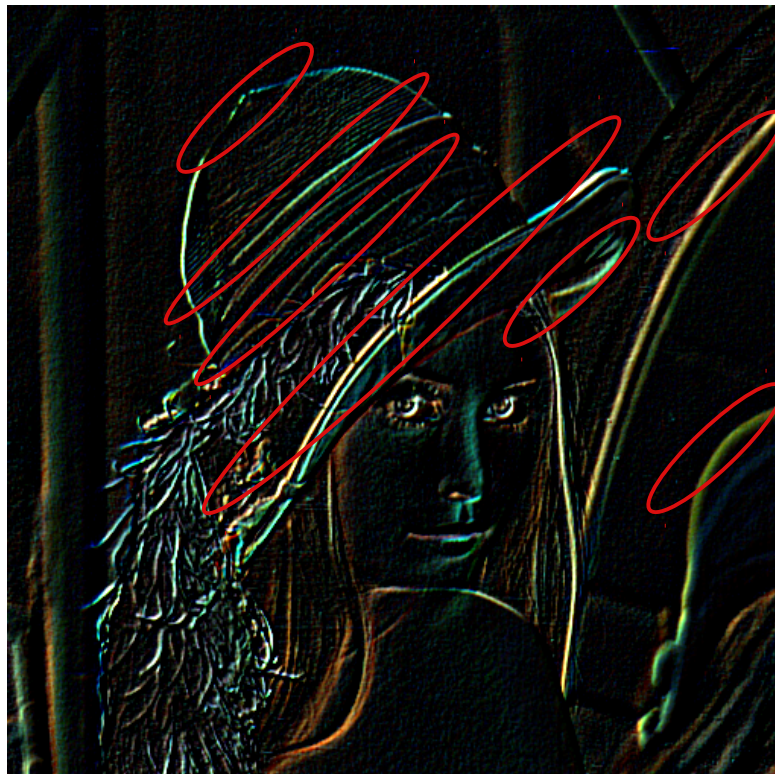
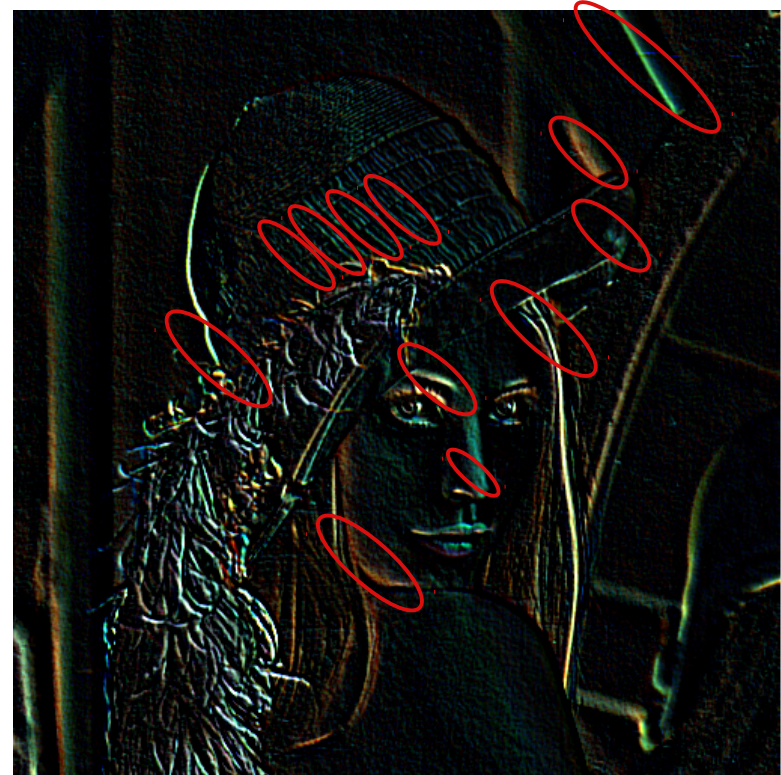
Horizontal Lines bigger
in Right image


Vertical Lines bigger
in Left image



Grad at different Orientations


$$\text{Gradient}_{45\text{deg}} = \begin{pmatrix} 0 & +1 & +2 \\ -1 & 0 & +1 \\ -2 & -1 & 0 \end{pmatrix}$$




$$\text{Gradient}_{-45\text{deg}} = \begin{pmatrix} -2 & -1 & 0 \\ -1 & 0 & +1 \\ 0 & +1 & +2 \end{pmatrix}$$

Gradient X (increasing Scale)

Detection of Vertical Edges




←

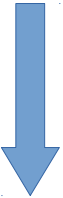
Small Scale :
Precise vertical edges position
Many small vertical edges
lot of Noise

→

Big Scale :
Imprecise edges position
Missed small vertical edges
lower Noise

Sobel Operator (~ Gradient)

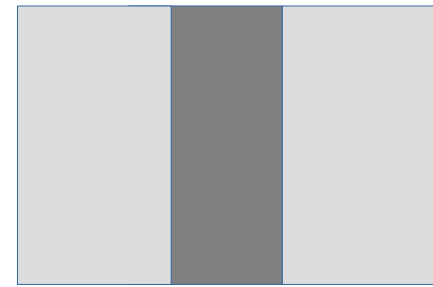

$$Sobel_x = \begin{pmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{pmatrix}$$


$$Sobel_y = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{pmatrix}$$

Second Derivatives ...

→ $Laplacian_x = \begin{pmatrix} -1 & +2 & -1 \\ -1 & +2 & -1 \\ -1 & +2 & -1 \end{pmatrix}$

Correlation = Detect changes



↓ $Laplacian_y = \begin{pmatrix} -1 & -1 & -1 \\ +2 & +2 & +2 \\ -1 & -1 & -1 \end{pmatrix}$

Correlation = Detect changes



Second Derivative Filters

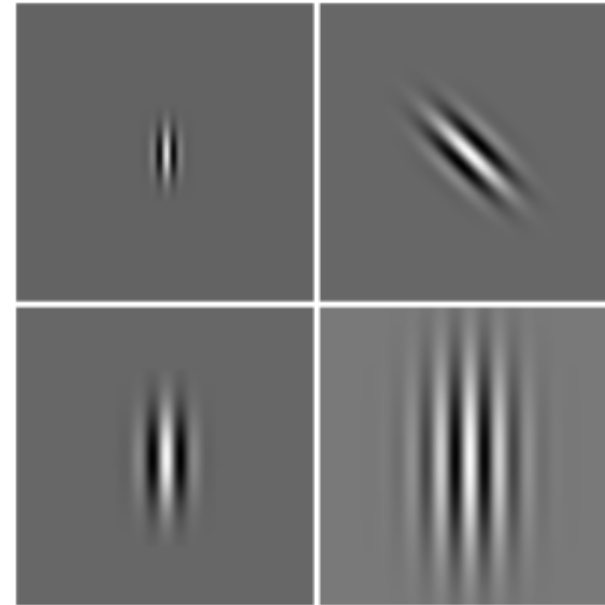
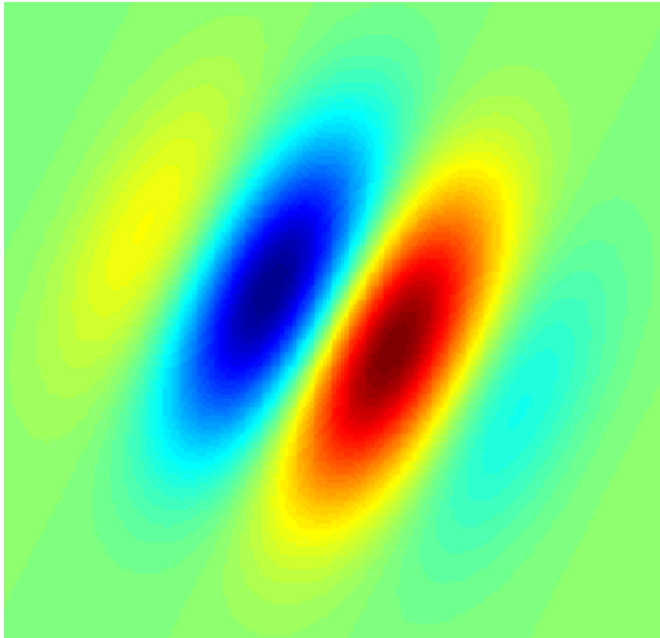
Example..

$$\begin{pmatrix} -1 & -1 & 0 & 4 & 0 & -1 & -1 \\ -1 & -1 & 0 & 4 & 0 & -1 & -1 \\ -1 & -1 & 0 & 4 & 0 & -1 & -1 \\ -1 & -1 & 0 & 4 & 0 & -1 & -1 \\ -1 & -1 & 0 & 4 & 0 & -1 & -1 \end{pmatrix}$$



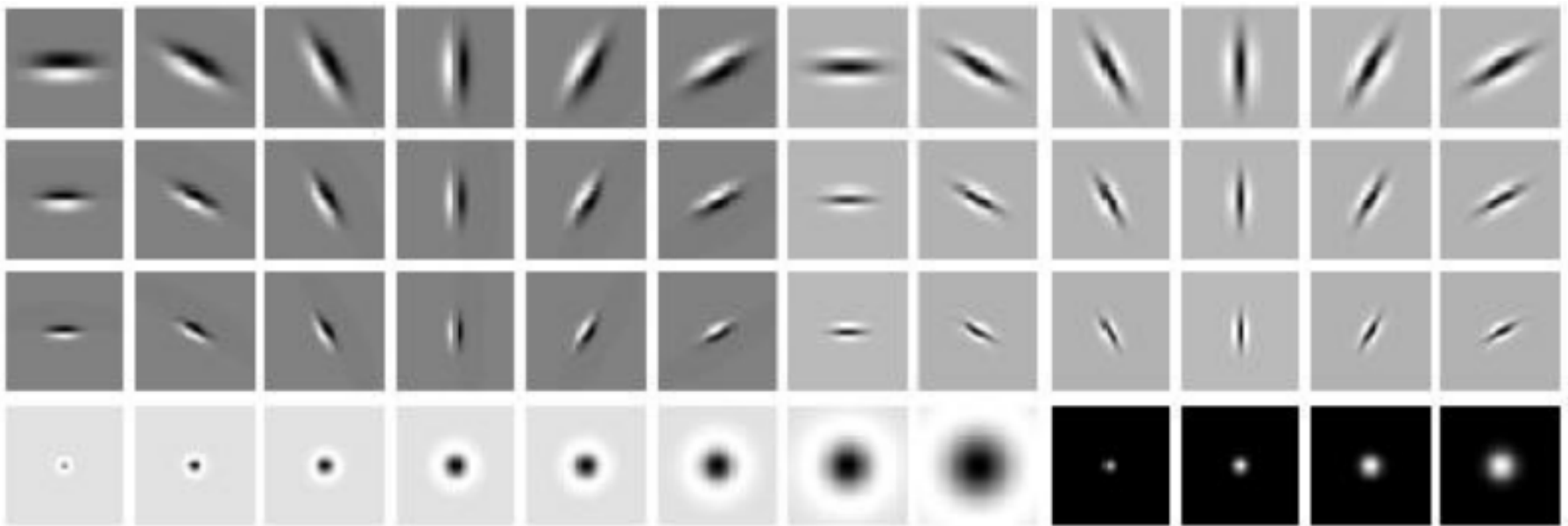
$$\begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Gabor Filter



Gabor filters are a “local” way of getting image frequency content
(idem Fourier coefficient... but local)
(idem Wavelet)

Family of Gaussian Filters (Leung-Malik Filter)



Cf next ... Same as Visual Cortex
in Biology !

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Applying Min , Median, Max ...instead of Linear Sums

Mathematical morphology

From Wikipedia, the free encyclopedia

Mathematical morphology (MM) is a theory and technique for the analysis and processing of geometrical structures, based on [set theory](#), [lattice theory](#), [topology](#), and [random functions](#). MM is most commonly applied to [digital images](#), but it can be employed as well on [graphs](#),

The basic morphological operators are [erosion](#), [dilation](#), [opening](#) and [closing](#).

MM was originally developed for [binary images](#), and was later extended to [grayscale functions](#) and images.

History [\[edit \]](#)

Mathematical Morphology was developed 1964 by the collaborative work of [Georges Matheron](#) and [Jean Serra](#), at the *École des Mines de Paris*, France. Matheron supervised the PhD thesis of Serra, devoted to the

Min (Dilate Black - Erode White)
/ Max(Dilate White – Erode Black)



Min-then-Max \neq Max-then-Min



Min-then-Max



Max-then-Min

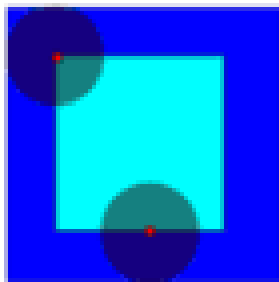
Filtering Out All Noise !!

Both OPEN and CLOSE Filter out small noise pixels

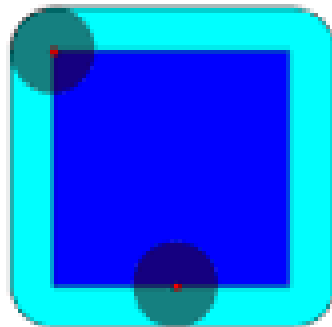
The image is looks similar to original...
(cf next for details)

Interpreting Kernel As Point Touching/Within Figure

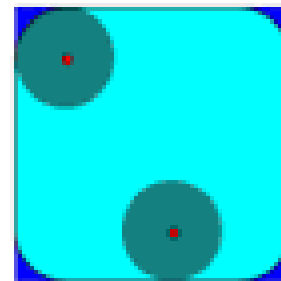
Erosion



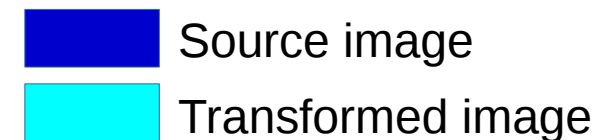
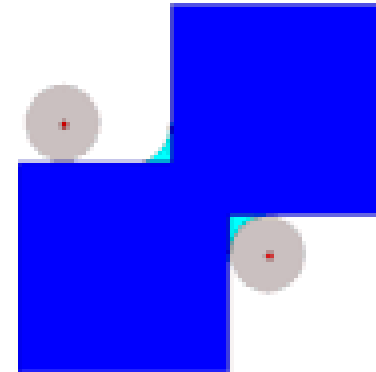
Dilation



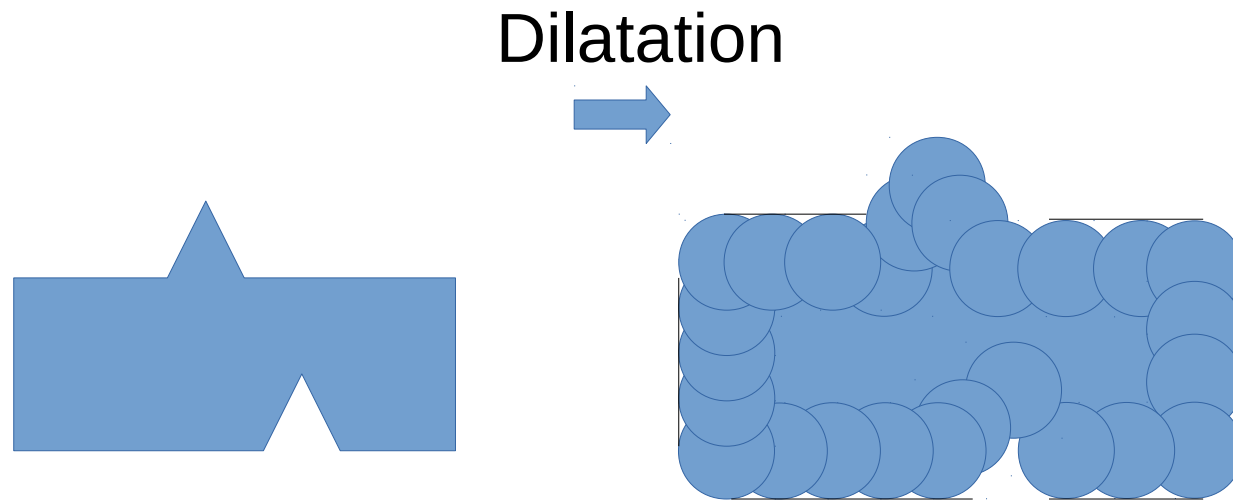
Opening



Closing



Dilatation (Kernel = Disk)



Erosion and Dilatation Kernel do not behave same on small Wholes and Bumps

Dilate-then-Erode \neq Erode-then-Dilate

Dilatation (Min)

Erosion(Max)

Erosion (Max)

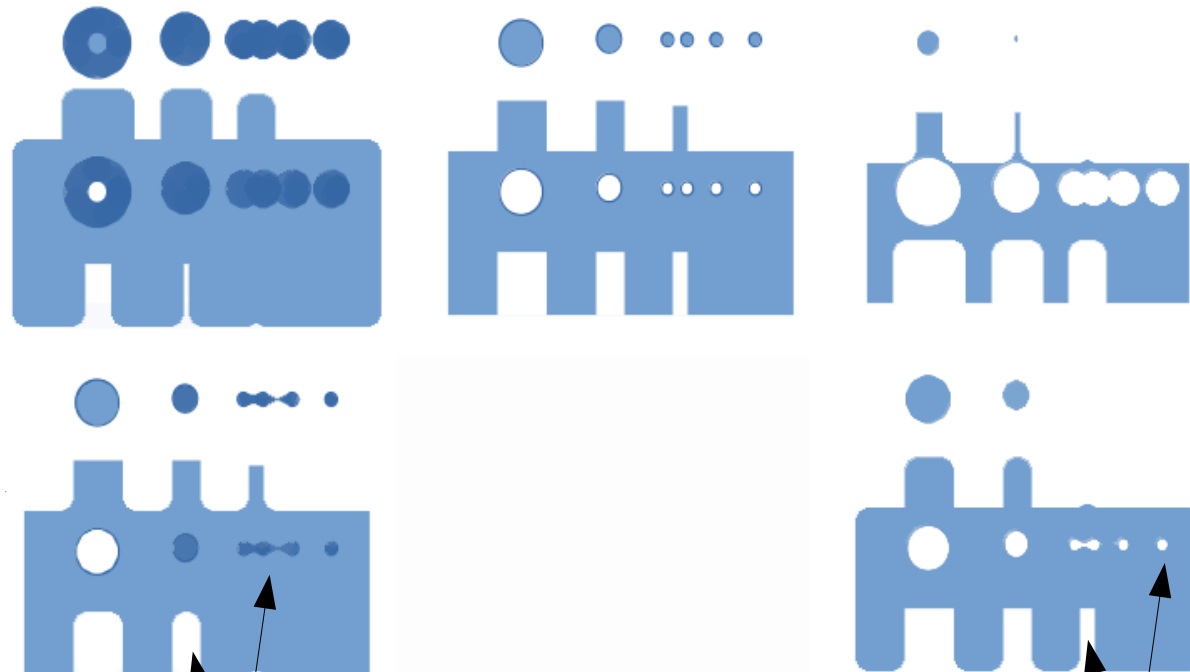
Dilatation (Min)

Dilatation-then-Erosion
= Close

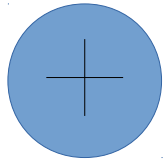
Erosion-Then-Dilatation
= Open

It closes wholes

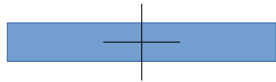
It keep wholes
round corner, remove small noises



Using Kernel Shape segment != disk



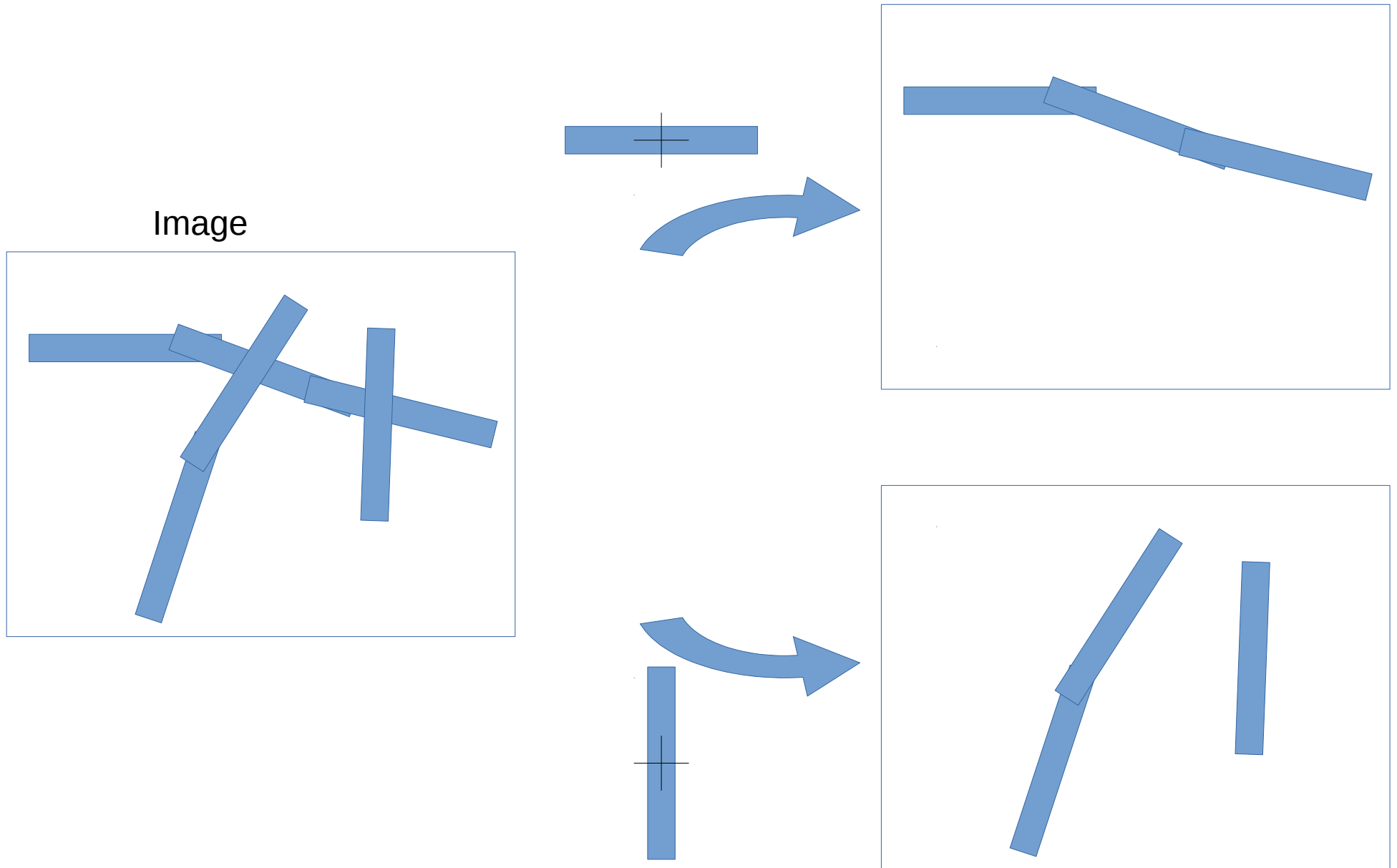
OK For detecting “filled” area
(... Whatever the direction)



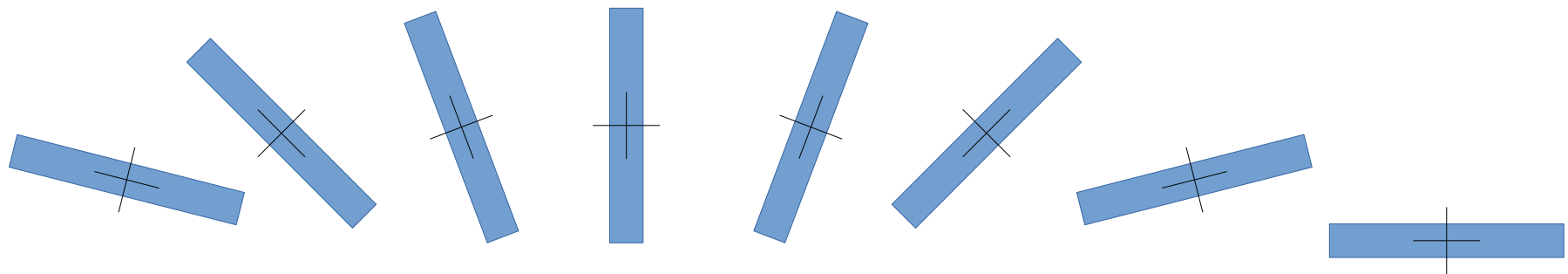
OK For detecting Horizontal segments!
(... very sensitive to direction)

Similar to GradY Detection for Linear Filters

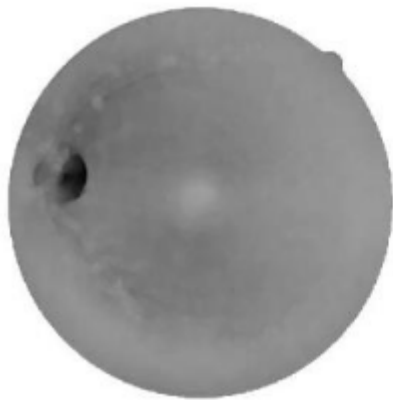
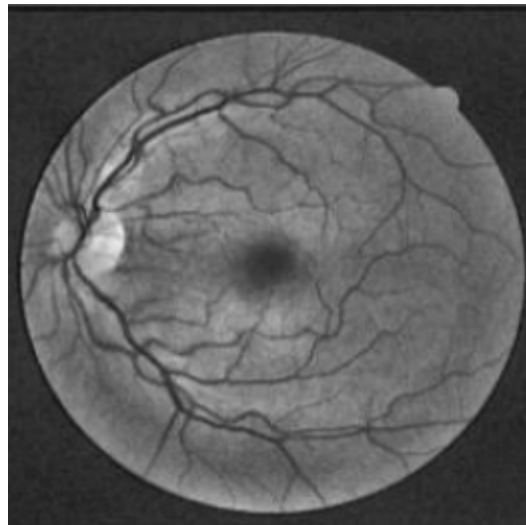
Example: Opening With Segment



Kernel Shapes Orientation



Example : Medical Image Blood Vessels (image of Eye)



Opening

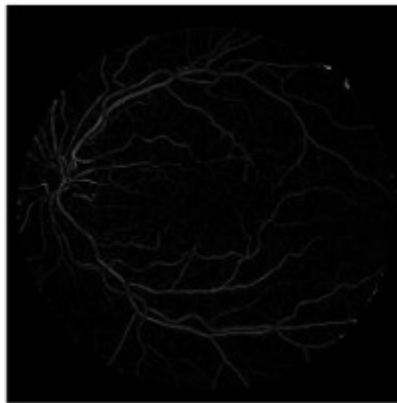
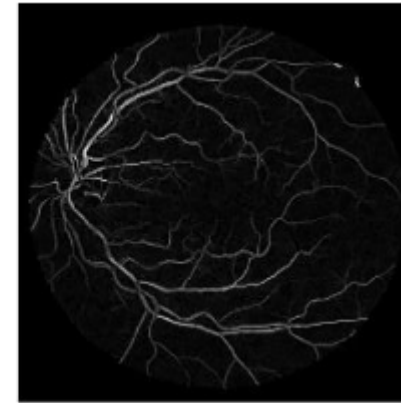


Image-Opening



Reconstruction by Erosion
summing All Segment Erosions
...

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Primary Visual Cortex V1

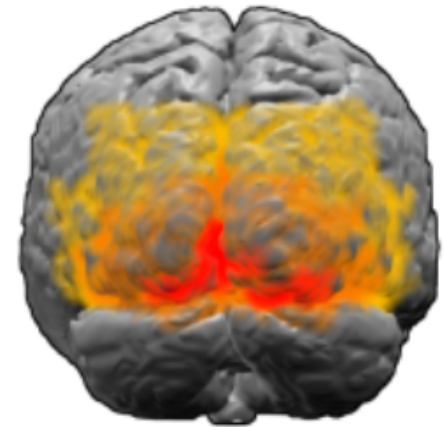
Visual cortex

From Wikipedia, the free encyclopedia

The **visual cortex** of the [brain](#) is a part of the [cerebral cortex](#) that plays an important role in processing [visual information](#). It is located in the [occipital lobe](#) in the back of the skull.

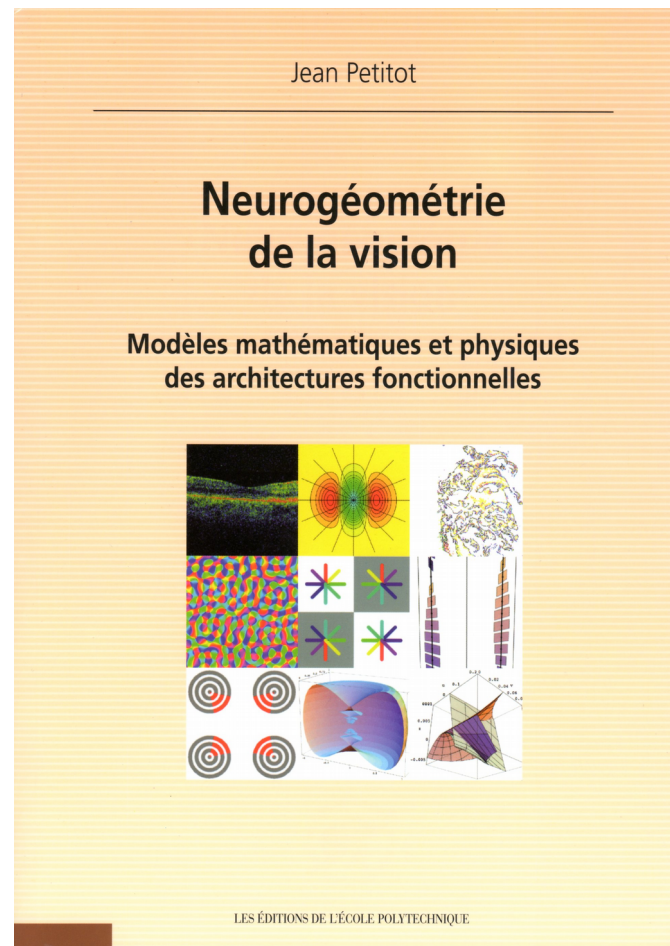
Visual information coming from the eye, goes through the [lateral geniculate nucleus](#), that is located in the [thalamus](#), and then reaches the visual cortex. The part of the visual cortex that receives the sensory inputs from the [thalamus](#) is the primary visual cortex, also known as **Visual area one (V1)**, and the **striate cortex**. The **extrastriate areas** consist of visual areas two (**V2**), three (**V3**), four (**V4**), and five (**V5**).^[1]

Visual cortex



Book: NeuroGeometry of Vision Jean Petitot

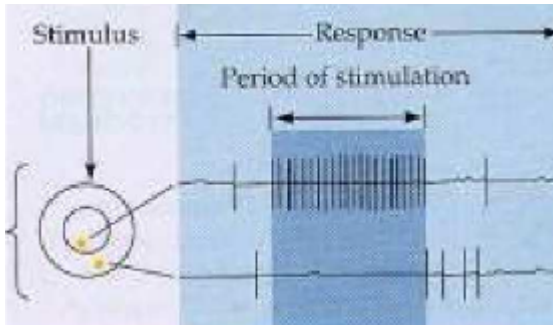
<http://jean.petitot.pagesperso-orange.fr/NGV.html>



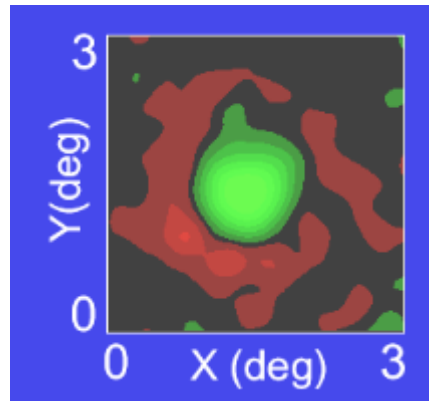
http://jeanpetitot.com/ArticlesPDF/Petitot_NGV_2008.pdf

Measuring Response of Neuron

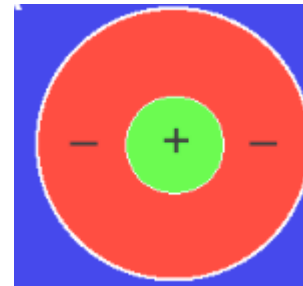
Experiment (on a cat...)



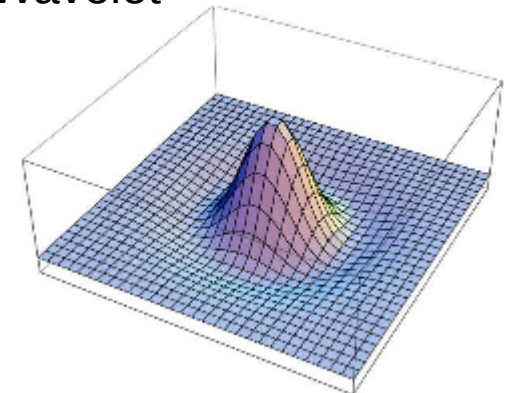
Measured
Response fonction



Idealised
Response fonction

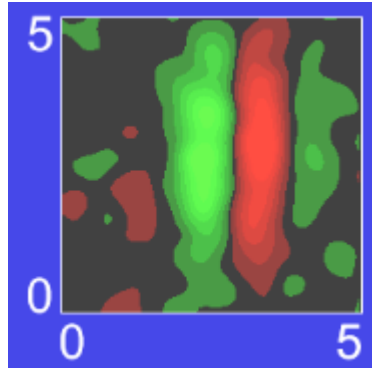


Math
Model (Gabor / Gaussian)
Wavelet

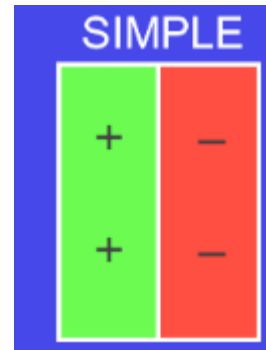


Other Neurons..

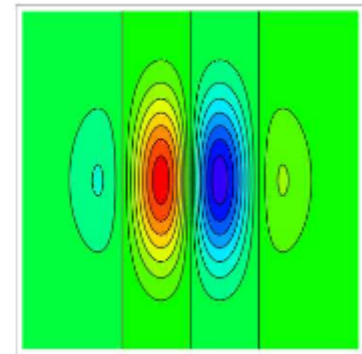
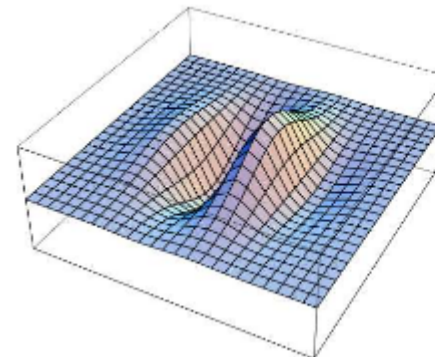
Measured
Response fonction



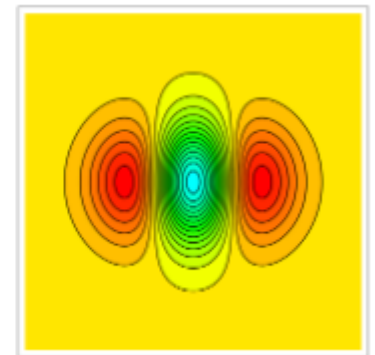
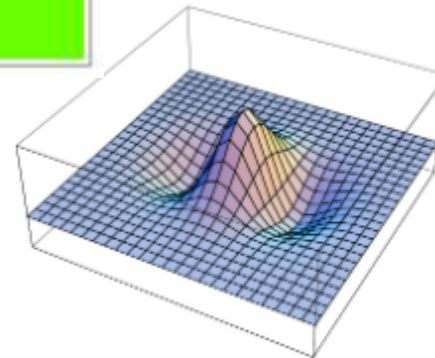
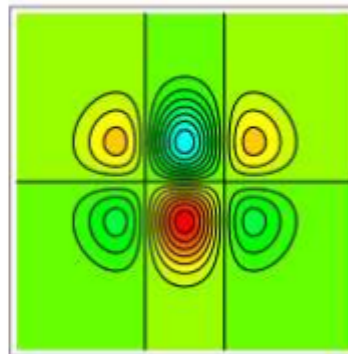
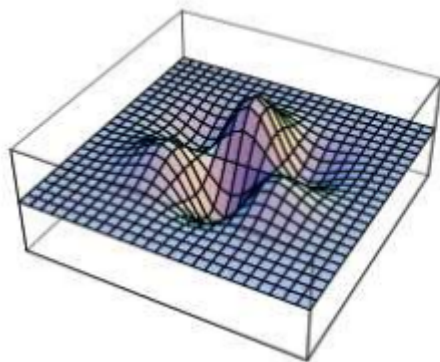
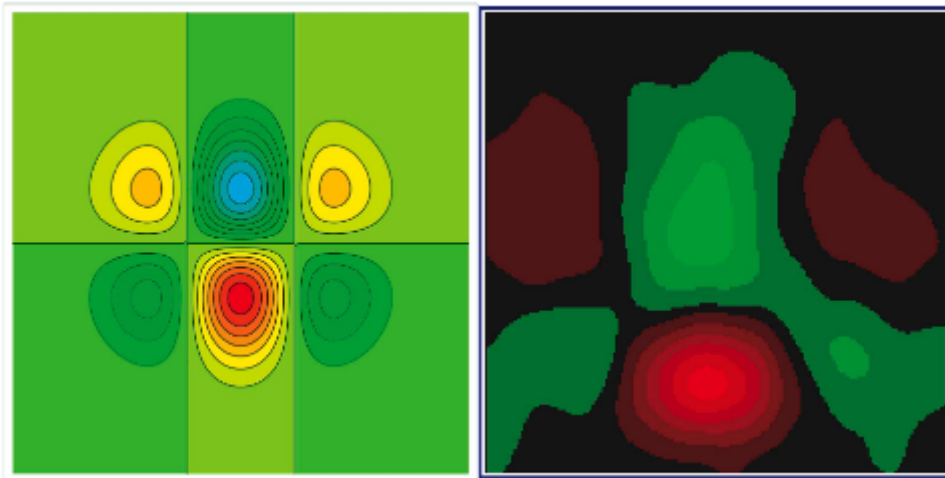
Idealised
Response fonction



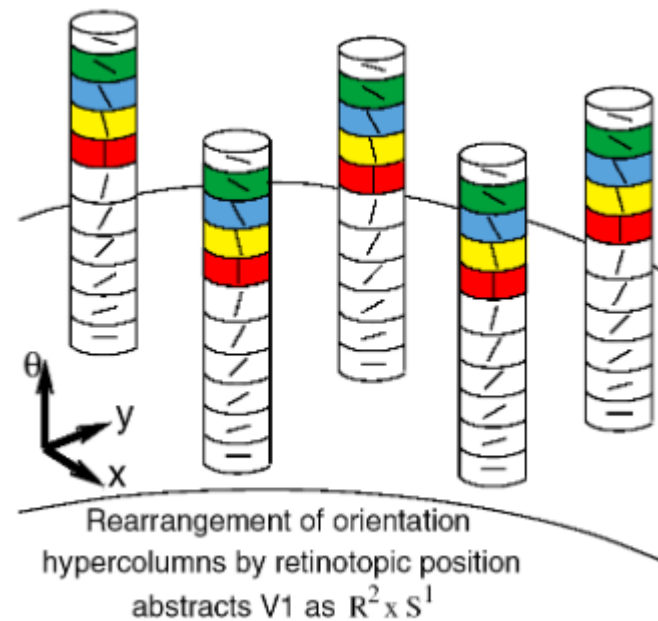
Math
Model (Gabor / Gaussian)
Wavelet



Other Neurons ...

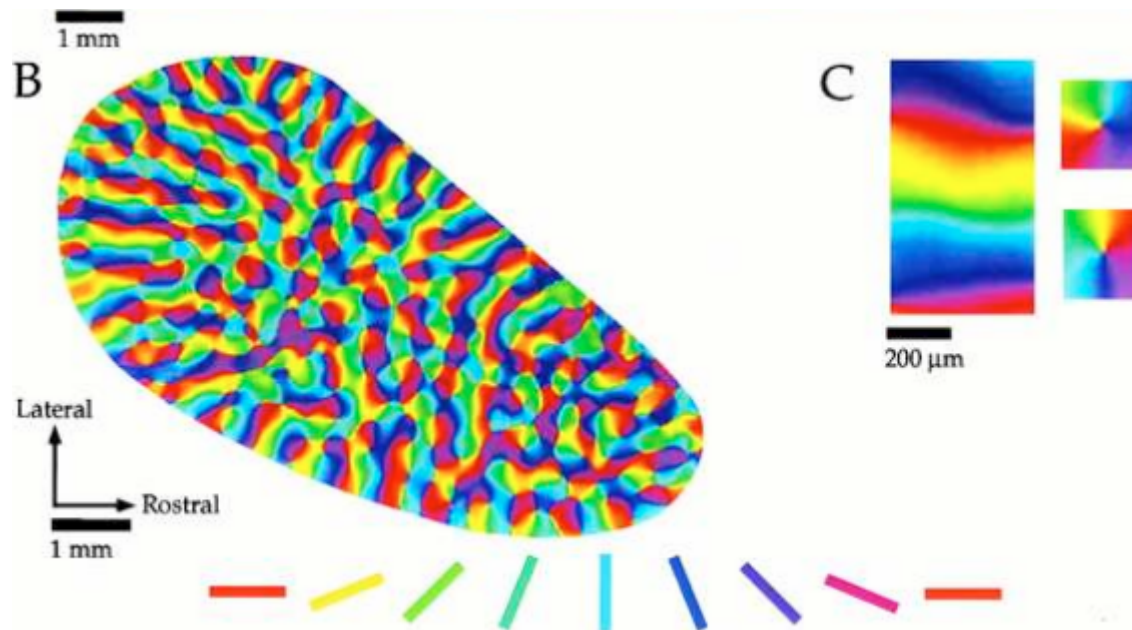


Visual Cortex = Parallel Computing For Wavelets...

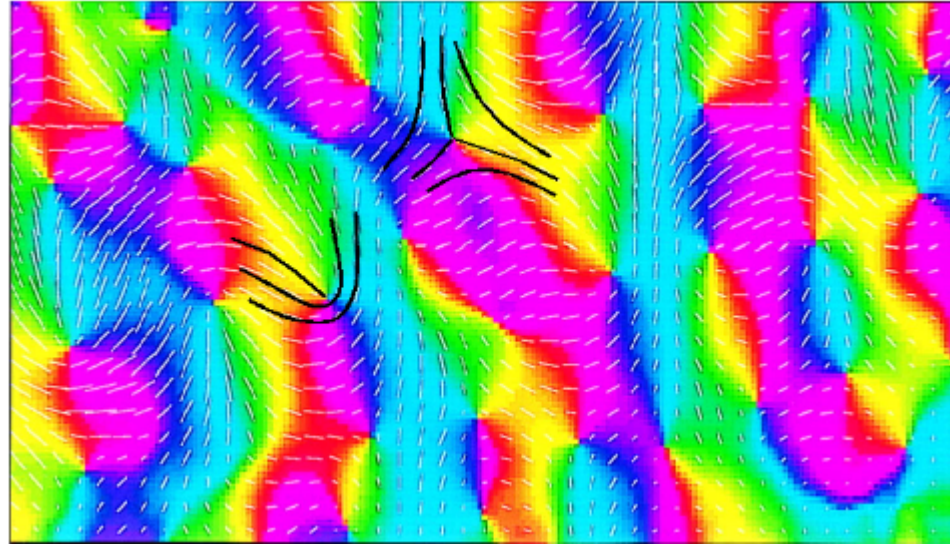


Visual Cortex 2D+Direction

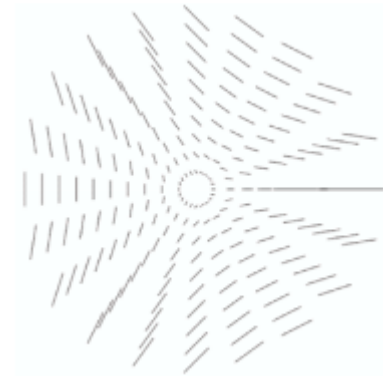
“Bio-Hardware implementation”



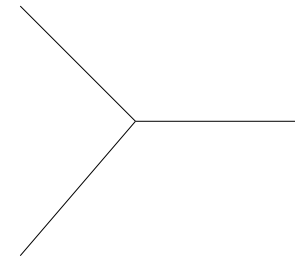
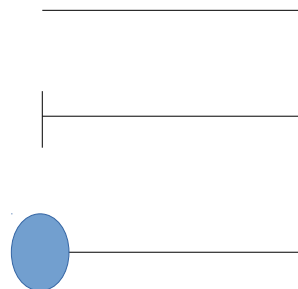
Oerientation Of PinWheels



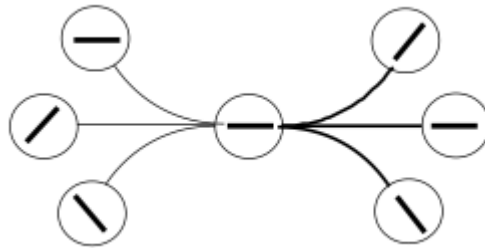
Detection for Special Points “End-Point” / “Corner” Point



Recognized
patterns:



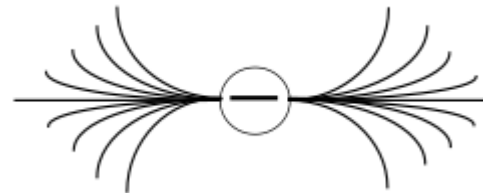
Co-Activation / Inhibition of Neurons



Bad continuation
=> **Inhibition**

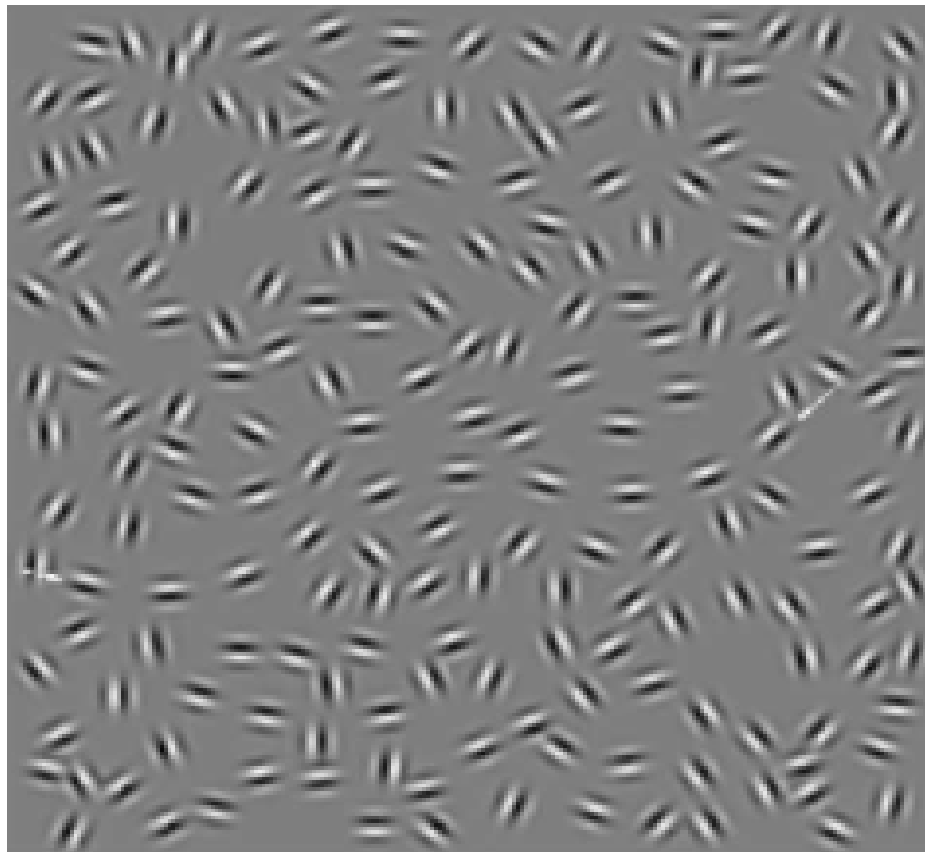


Good continuation
=> **activation**



Example of Neuron Co-Activation “good continuation” criteria

Do you see a Line ?



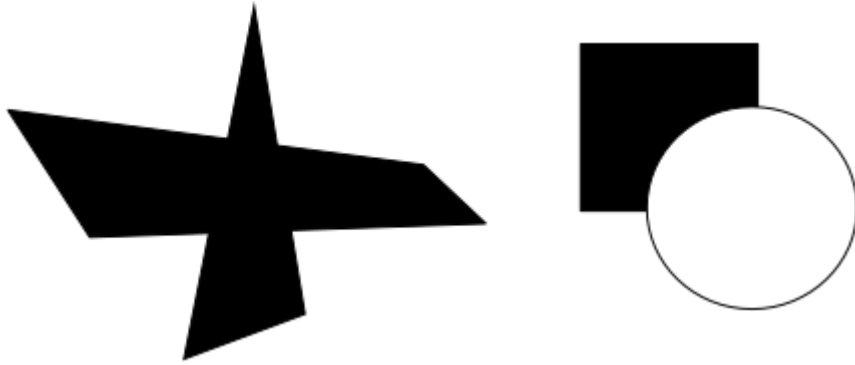
Neon Effect

The inside is perfectly white ...
But it looks ilighted by red



Illusory Contours

Impression of 3D (shape over other)

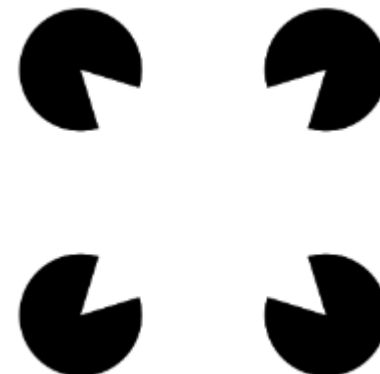
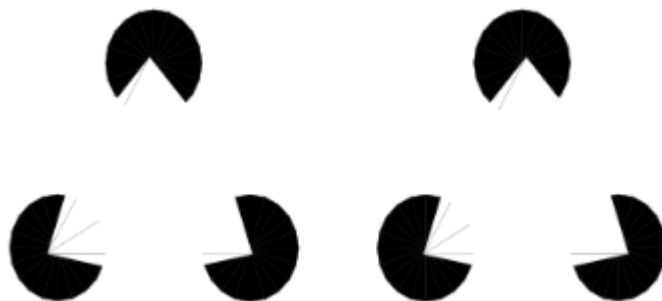


Illusory Kaniksa Triangles

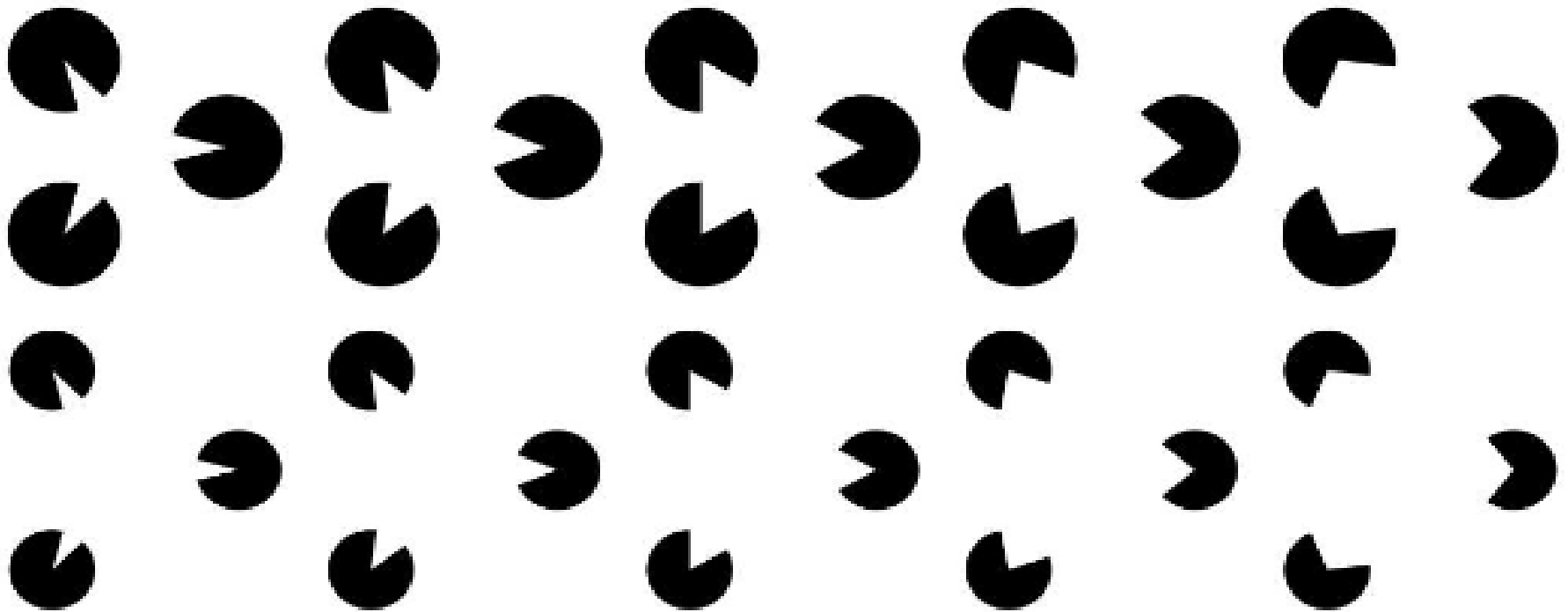
Illusory "Triangle" over circles



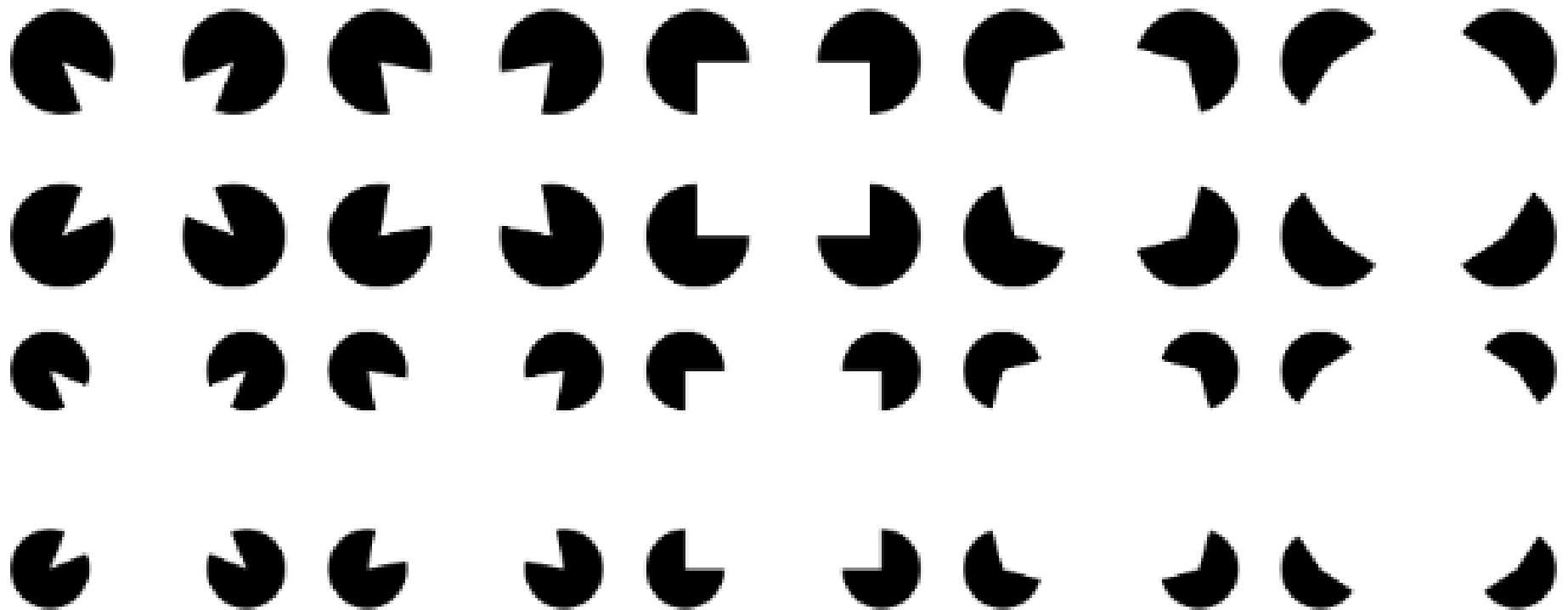
Illusory "Curved" square over circles



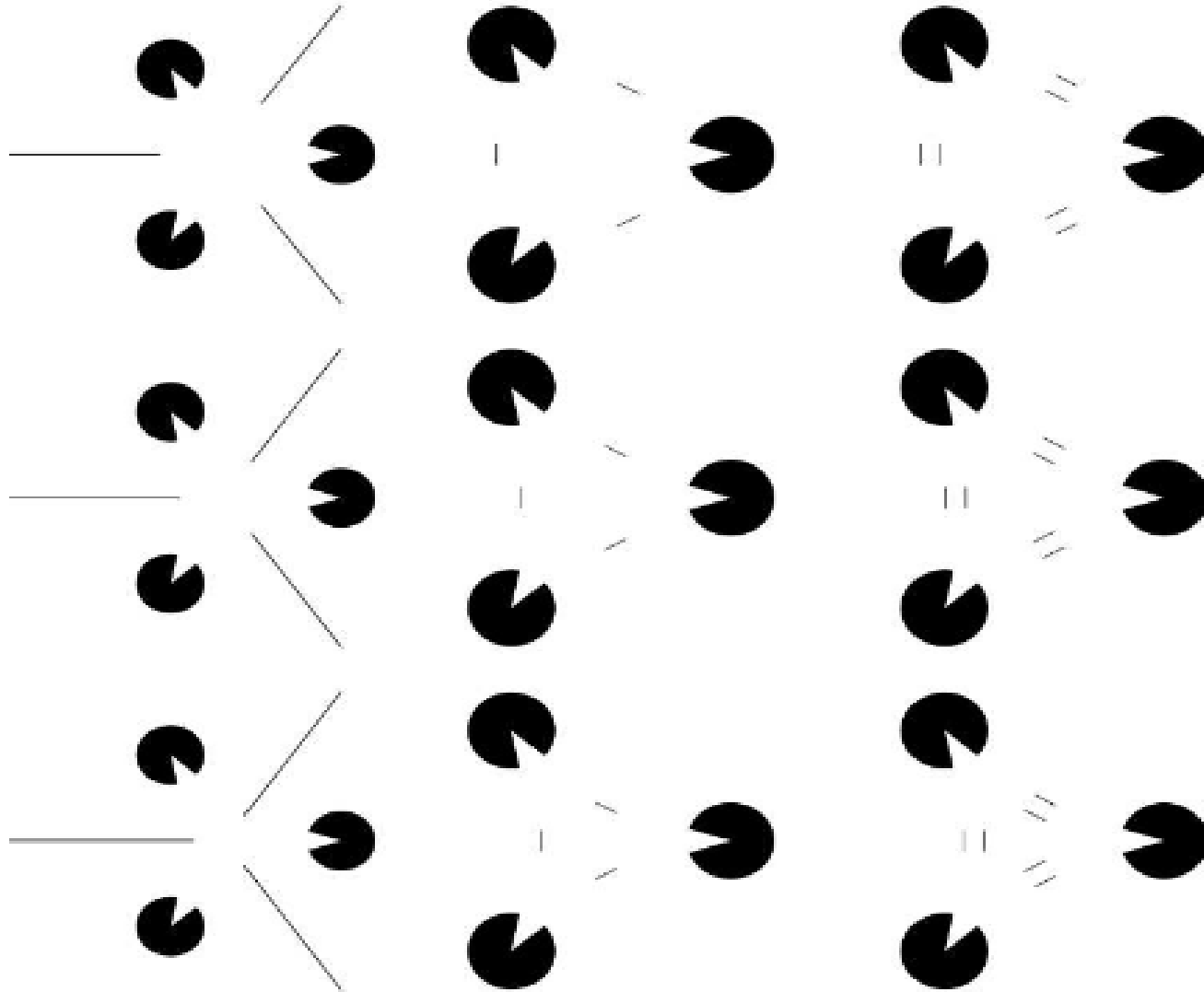
Illusory Kanizsa Triangle ...Up to limit Angle



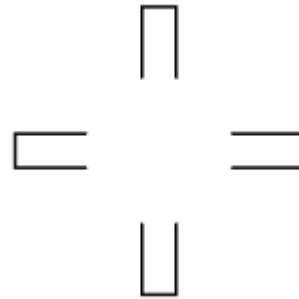
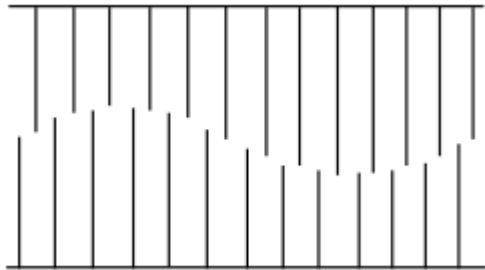
Illusory Kaniksa squares..



Kaniksa illusions re-inforced by lines



Illusory Line



Illusory "Circle" or "Square"
(oscillating)

Outline

Linear Filter

Non-Linear Filter : Mathematical Morphology
Erode/Dilate, Open/Close

Visual Cortex – bio mimetism

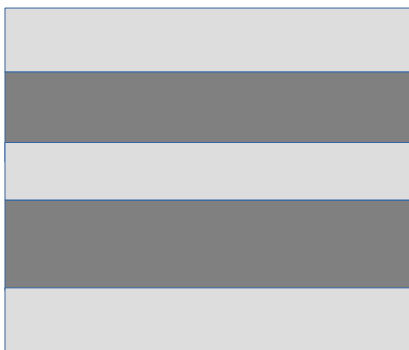
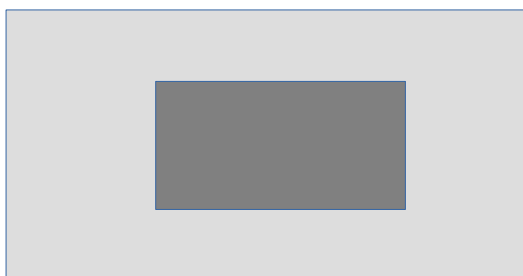
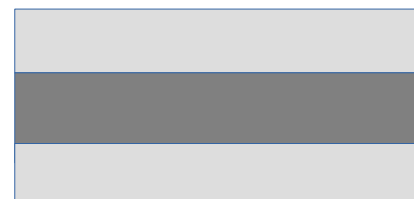
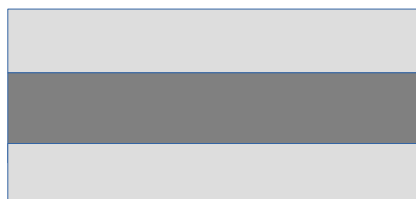
Object Recognition, Segmentation, Optical Flow, ..

Deep Neural Networks

Pattern Matching

Optimise Pattern Matching
= Find Max Correlation

Correlation using Combinations of Rectangular Masks



Integral Image

FAST Compute Correlation for Rectangular Maks

Viola Jones Algorithm

Outline

Linear Filter

Non-Linear Filter : Mathematical Morphology
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Deep Neural Networks

